CHAPTER 6

Sequences and Series

Section-A JEE Advanced/ IIT-JEE

A Fill in the Blanks

1. The sum of integers from 1 to 100 that are divisible by 2 or 5 is .... *(1984 - 2 Marks)*
2. The solution of the equation $\log_7 \log_5(\sqrt{x+5}+\sqrt{x})=0$ is .... *(1986 - 2 Marks)*
3. The sum of the first n terms of the series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+$......... is $n\,(n+1)^2/2$, when n is even. When n is odd, the sum is .... *(1988 - 2 Marks)*
4. Let the harmonic mean and geometric mean of two positive numbers be the ratio 4 : 5. Then the two number are in the ratio .... *(1992 - 2 Marks)*
5. For any odd integer n $\geq$ 1, $n^3-(n-1)^3+...+(-1)^{n-1}\,1^3=$ .... *(1996 - 1 Mark)*
6. Let p and q be roots of the equation $x^2-2x+A=0$ and let r and s be the roots of the equation $x^2-18x+B=0$. If $p<q<r<s$ are in arithmetic progression, then $A=$ .... and $B=$............. *(1997 - 2 Marks)*

C MCQs with One Correct Answer

1. If x, y and z are pth, qth and rth terms respectively of an A.P. and also of a G.P., then $x^{y-z}y^{z-x}z^{x-y}$ is equal to :
   (a) xyz (b) 0 (c) 1 (d) None of these
2. The third term of a geometric progression is 4. The product of the first five terms is *(1982 - 2 Marks)*
   (a) 4^3 (b) 4^5 (c) 4^4 (d) none of these
3. The rational number, which equals the number $2.\overline{357}$ with recurring decimal is *(1983 - 1 Mark)*
   (a) $\frac{2355}{1001}$ (b) $\frac{2379}{997}$ (c) $\frac{2355}{999}$ (d) none of these
4. If a, b, c are in G.P., then the equations $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root if $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in — *(1985 - 2 Marks)*
   (a) A.P. (b) G.P. (c) H.P. (d) none of these
5. Sum of the first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+$............. is equal to *(1988 - 2 Marks)*
   (a) 2^n-n-1 (b) $1-2^{-n}$
   (c) $n+2^{-n}-1$ (d) 2^n+1.
6. The number $\log_2 7$ is *(1990 - 2 Marks)*
   (a) an integer (b) a rational number
   (c) an irrational number (d) a prime number
7. If $ln(a+c), ln\,(a-c), ln\,(a-2b+c)$ are in A.P., then *(1994)*
   (a) a, b, c are in A.P. (b) a^2, b^2, c^2 are in A.P.
   (c) a, b, c are in G.P. (d) a, b, c are in H.P.
8. Let $a_1, a_2, ...\, a_{10}$ be in A, P, and $h_1, h_2,....h_{10}$ be in H.P. If $a_1=h_1=2$ and $a_{10}=h_{10}=3$, then a_4h_7 is *(1999 - 2 Marks)*
   (a) 2 (b) 3 (c) 5 (d) 6
9. The harmonic mean of the roots of the equation $\left(5+\sqrt{2}\right)x^2-\left(4+\sqrt{5}\right)x+8+2\sqrt{5}=0$ is *(1999 - 2 Marks)*
   (a) 2 (b) 4 (c) 6 (d) 8
10. Consider an infinite geometric series with first term a and common ratio r. If its sum is 4 and the second term is 3/4, then *(2000S)*
    (a) $a=\frac{4}{7}, r=\frac{3}{7}$ (b) $a=2, r=\frac{3}{8}$
    (c) $a=\frac{3}{2}, r=\frac{1}{2}$ (d) $a=3, r=\frac{1}{4}$
11. Let α, β be the roots of $x^2-x+p=0$ and γ, δ be the roots of $x^2-4x+q=0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q respectively, are *(2001S)*
    (a) $-2,-32$ (b) $-2, 3$ (c) $-6, 3$ (d) $-6,-32$
12. Let the positive numbers a, b, c, d be in A.P. Then abc, abd, acd, bcd are *(2001S)*
    (a) NOT in A.P./G.P./H.P. (b) in A.P.
    (c) in G.P. (d) in H.P.
13. If the sum of the first $2n$ terms of the A.P. 2, 5, 8, …, is equal to the sum of the first n terms of the A.P. 57, 59, 61, …, then n equals *(2001S)*
    (a) 10 (b) 12 (c) 11 (d) 13
14. Suppose a, b, c are in A.P. and a^2, b^2, c^2 are in G.P. if $a<b<c$ and $a+b+c=\frac{3}{2}$, then the value of a is *(2002S)*
    (a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2\sqrt{3}}$ (c) $\frac{1}{2}-\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}-\frac{1}{\sqrt{2}}$
15. An infinite G.P. has first term 'x' and sum '5', then x belongs to *(2004S)*
    (a) $x<-10$ (b) $-10<x<0$
    (c) $0<x<10$ (d) $x>10$
16. In the quadratic equation $ax^2+bx+c=0, \Delta=b^2-4ac$ and $\alpha+\beta, \alpha^2+\beta^2, \alpha^3+\beta^3$, are in G.P. where α, β are the root of $ax^2+bx+c=0$, then *(2005S)*
    (a) $\Delta\neq 0$ (b) $b\Delta=0$ (c) $c\Delta=0$ (d) $\Delta=0$

17. In the sum of first n terms of an A.P. is cn^2, then the sum of squares of these n terms is *(2009)*

(a) $\frac{n(4n^2-1)c^2}{6}$ (b) $\frac{n(4n^2+1)c^2}{3}$

(c) $\frac{n(4n^2-1)c^2}{3}$ (d) $\frac{n(4n^2+1)c^2}{6}$

18. Let $a_1, a_2, a_3, ...$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is *(2012)*
(a) 22 (b) 23 (c) 24 (d) 25

19. Let $b_i > 1$ for $i = 1, 2, ..., 101$. Suppose $\log_e b_1, \log_e b_2, ..., \log_e b_{101}$ are in Arithmetic Progression (A.P.) with the common difference $\log_e 2$. Suppose $a_1, a_2, ..., a_{101}$ are in A.P. such that $a_1 = b_1$ and $a_{51} = b_{51}$. If $t = b_1 + b_2 + ... + b_{51}$ and $s = a_1 + a_2 + ... + a5_3$, then *(JEE Adv. 2016)*
(a) $s > t$ and $a_{101} > b_{101}$ (b) $s > t$ and $a_{101} < b_{101}$
(c) $s < t$ and $a_{101} > b_{101}$ (d) $s < t$ and $a_{101} < b_{101}$

D MCQs with One or More than One Correct

1. If the first and the $(2n-1)$st terms of an A.P., a G.P. and an H.P. are equal and their n-th terms are a, b and c respectively, then *(1988 - 2 Marks)*
(a) $a = b = c$ (b) $a \geq b \geq c$
(c) $a + c = b$ (d) $ac - b^2 = 0.$

2. For $0 < \phi < \pi/2$, if

$$x = \sum_{n=0}^{\infty} \cos^{2n}\phi, \; y = \sum_{n=0}^{\infty} \sin^{2n}\phi, \; z = \sum_{n=0}^{\infty} \cos^{2n}\phi \sin^{2n}\phi$$

then: *(1993 - 2 Marks)*
(a) $xyz = xz + y$ (b) $xyz = xy + z$
(c) $xyz = x + y + z$ (d) $xyz = yz + x$

3. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then *(1998 - 2 Marks)*
(a) $b_0 = 1, b_1 = 3$ (b) $b_0 = 0, b_1 = n$
(c) $b_0 = -1, b_1 = n$ (d) $b_0 = 0, b_1 = n^2 - 3n + 3$

4. Let T_r be the r^{th} term of an A.P., for $r = 1, 2, 3, ...$ If for some positive integers m, n we have

$T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$, then T_{mn} equals *(1998 - 2 Marks)*

(a) $\frac{1}{mn}$ (b) $\frac{1}{m}+\frac{1}{n}$ (c) 1 (d) 0

5. If $x > 1, y > 1, z > 1$ are in G.P., then $\frac{1}{1+\ln x}, \frac{1}{1+\ln y}, \frac{1}{1+\ln z}$ are in *(1998 - 2 Marks)*
(a) A.P. (b) H.P. (c) GP (d) None of these

6. For a positive integer n, let

$a(n) = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + ... \frac{1}{(2^n)-1}$. Then *(1999 - 3 Marks)*
(a) $a(100) \leq 100$ (b) $a(100) > 100$
(c) $a(200) \leq 100$ (d) $a(200) > 100$

7. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then *(2008)*

(a) $\frac{1}{PS}+\frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$ (b) $\frac{1}{PS}+\frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$

(c) $\frac{1}{PS}+\frac{1}{ST} < \frac{4}{QR}$ (d) $\frac{1}{PS}+\frac{1}{ST} > \frac{4}{QR}$

8. Let $S_n = \sum_{k=1}^{n} \frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2+kn+k^2}$ for $n = 1, 2, 3, ...$ Then, *(2008)*

(a) $S_n < \frac{\pi}{3\sqrt{3}}$ (b) $S_n > \frac{\pi}{3\sqrt{3}}$

(c) $T_n < \frac{\pi}{3\sqrt{3}}$ (d) $T_n > \frac{\pi}{3\sqrt{3}}$

9. Let $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$. Then S_n can take value(s) *(JEE Adv. 2013)*

(a) 1056 (b) 1088 (c) 1120 (d) 1332

E Subjective Problems

1. The harmonic mean of two numbers is 4. Their arithmetic mean A and the geometric mean G satisfy the relation.
$2A + G^2 = 27$
Find the two numbers. *(1979)*

2. The interior angles of a polygon are in arithmetic progression. The smallest angle is 120°, and the common difference is 5°, Find the number of sides of the polygon. *(1980)*

3. Does there exist a geometric progression containing 27, 8 and 12 as three of its terms ? If it exits, how many such progressions are possible ? *(1982 - 3 Marks)*

4. Find three numbers a, b, c, between 2 and 18 such that
(i) their sum is 25
(ii) the numbers 2, a, b sare consecutive terms of an A.P. and
(iii) the numbers b, c, 18 are consecutive terms of a G.P. *(1983 - 2 Marks)*

5. If $a > 0, b > 0$ and $c > 0$, prove that

$(a + b + c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$ *(1984 - 2 Marks)*

6. If n is a natural number such that

$n = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot p_3^{\alpha_3} .... p_k^{\alpha_k}$ and $p_1, p_2, ..., p_k$ are distinct primes, then show that $\ln n \geq k \ln 2$ *(1984 - 2 Marks)*

7. Find the sum of the series :

$\sum_{r=0}^{n} (-1)^r \, {}^nC_r [\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}......$ up to m terms]

(1985 - 5 Marks)

8. Solve for x the following equation : ***(1987 - 3 Marks)***

$$\log_{(2x+3)}(6x^2+23x+21) = 4-\log_{(3x+7)}(4x^2+12x+9)$$

9. If $\log_3 2$, $\log_3(2^x-5)$, and $\log_3\left(2^x-\frac{7}{2}\right)$ are in arithmetic progression, determine the value of x. ***(1990 - 4 Marks)***

10. Let p be the first of the n arithmetic means between two numbers and q the first of n harmonic means between the same numbers. Show that q does not lie between p and $\left(\frac{n+1}{n-1}\right)^2 p$. ***(1991 - 4 Marks)***

11. If $S_1, S_2, S_3, ..., S_n$ are the sums of infinite geometric series whose first terms are 1, 2, 3, ..., n and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, ... $\frac{1}{n+1}$ respectively, then find the values of $S_1^2+S_2^2+S_3^2+ .... + S_{2n-1}^2$ ***(1991 - 4 Marks)***

12. The real numbers x_1, x_2, x_3 satisfying the equation $x^3-x^2+\beta x+\gamma=0$ are in AP. Find the intervals in which β and γ lie. ***(1996 - 3 Marks)***

13. Let a, b, c, d be real numbers in G.P. If u, v, w, satisfy the system of equations ***(1999 - 10 Marks)***

$$u+2v+3w=6$$
$$4u+5v+6w=12$$
$$6u+9v=4$$

then show that the roots of the equation

$$\left(\frac{1}{u}+\frac{1}{v}+\frac{1}{w}\right)x^2 + [(b-c)^2+(c-a)^2+(d-b)^2]\,x+u+v+w=0$$

and $20x^2+10(a-d)^2 x-9=0$ are reciprocals of each other.

14. The fourth power of the common difference of an arithmetic progression with integer entries is added to the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer. ***(2000 - 4 Marks)***

15. Let $a_1, a_2, ..., a_n$ be positive real numbers in geometric progression. For each n, let A_n, G_n, H_n be respectively, the arithmetic mean, geometric mean, and harmonic mean of $a_1, a_2, ..., a_n$. Find an expression for the geometric mean of $G_1, G_2, ..., G_n$ in terms of $A_1, A_2, ..., A_n, H_1, H_2, ..., H_n$. ***(2001 - 5 Marks)***

16. Let a, b be positive real numbers. If a, A_1, A_2, b are in arithmetic progression, a, G_1, G_2, b are in geometric progression and a, H_1, H_2, b are in harmonic progression, show that $\frac{G_1G_2}{H_1H_2}=\frac{A_1+A_2}{H_1+H_2}=\frac{(2a+b)(a+2b)}{9ab}$. ***(2002 - 5 Marks)***

17. If a, b, c are in A.P., a^2, b^2, c^2 are in H.P., then prove that either $a=b=c$ or $a, b, -\frac{c}{2}$ form a G.P. ***(2003 - 4 Marks)***

18. If $a_n=\frac{3}{4}-\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3+.....(-1)^{n-1}\left(\frac{3}{4}\right)^n$ and $b_n=1-a_n$, then find the least natural number n_0 such that $b_n>a_n\ \forall\, n\geq n_0$. ***(2006 - 6M)***

G Comprehension Based Questions

PASSAGE - 1

Let V_r denote the sum of first r terms of an arithmetic progression (A.P.) whose first term is r and the common difference is $(2r-1)$. Let $T_r=V_{r+1}-V_r-2$ and $Q_r=T_{r+1}-T_r$ for $r=1, 2, ...$

1. The sum $V_1+V_2+...+V_n$ is ***(2007 -4 marks)***

(a) $\frac{1}{12}n(n+1)(3n^2-n+1)$ (b) $\frac{1}{12}n(n+1)(3n^2+n+2)$

(c) $\frac{1}{2}n(2n^2-n+1)$ (d) $\frac{1}{3}(2n^3-2n+3)$

2. T_r is always ***(2007 -4 marks)***

(a) an odd number (b) an even number

(c) a prime number (d) a composite number

3. Which one of the following is a correct statement ? ***(2007 -4 marks)***

(a) $Q_1, Q_2, Q_3, ...$ are in A.P. with common difference 5

(b) $Q_1, Q_2, Q_3, ...$ are in A.P. with common difference 6

(c) $Q_1, Q_2, Q_3, ...$ are in A.P. with common difference 11

(d) $Q_1=Q_2=Q_3=....$

PASSAGE -2

Let A_1, G_1, H_1 denote the arithmetic, geometric and harmonic means, respectively, of two distinct positive numbers. For $n \geq 2$, Let A_{n-1} and H_{n-1} have arithmetic, geometric and harmonic means as A_n, G_n, H_n respectively.

4. Which one of the following statements is correct ? ***(2007 -4 marks)***

(a) $G_1>G_2>G_3>...$

(b) $G_1<G_2<G_3<...$

(c) $G_1=G_2=G_3=...$

(d) $G_1<G_3<G_5<...$ and $G_2>G_4>G_6>....$

5. Which one of the following statements is correct ? ***(2007 -4 marks)***

(a) $A_1>A_2>A_3>...$

(b) $A_1<A_2<A_3<...$

(c) $A_1>A_3>A_5>...$ and $A_2<A_4<A_6<...$

(d) $A_1<A_3<A_5<...$ and $A_2>A_4>A_6>...$

6. Which one of the following statements is correct ? ***(2007 -4 marks)***

(a) $H_1>H_2>H_3>...$

(b) $H_1<H_2<H_3<...$

(c) $H_1>H_3>H_5>...$ and $H_2<H_4<H_6<...$

(d) $H_1<H_3<H_6<...$ and $H_2>H_4>H_6>...$

GP_3480

H Assertion & Reason Type Questions

1. Suppose four distinct positive numbers a_1, a_2, a_3, a_4 are in G.P. Let $b_1 = a_1, b_2 = b_1 + a_2, b_3 = b_2 + a_3$ and $b_4 = b_3 + a_4$.
STATEMENT - 1 : The numbers b_1, b_2, b_3, b_4 are neither in A.P. nor in G.P.
and
STATEMENT - 2 : The numbers b_1, b_2, b_3, b_4 are in H.P. *(2008)*
(a) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is a correct explanation for STATEMENT - 1
(b) STATEMENT - 1 is True, STATEMENT - 2 is True; STATEMENT - 2 is **NOT** a correct explanation for STATEMENT - 1
(c) STATEMENT - 1 is True, STATEMENT - 2 is False
(d) STATEMENT - 1 is False, STATEMENT - 2 is True

I Integer Value Correct Type

1. Let S_k, $k = 1, 2, \ldots, 100$, denote the sum of the infinite geometric series whose first term is $\frac{k-1}{k!}$ and the common ratio is $\frac{1}{k}$. Then the value of $\frac{100^2}{100!} + \sum_{k=1}^{100} |(k^2 - 3k + 1)S_k|$ is *(2010)*

2. Let $a_1, a_2, a_3, \ldots, a_{11}$ be real numbers satisfying $a_1 = 15$, $27 - 2a_2 > 0$ and $a_k = 2a_{k-1} - a_{k-2}$ for $k = 3, 4, \ldots, 11$.
if $\frac{a_1^2 + a_2^2 + \ldots + a_{11}^2}{11} = 90$, then the value of $\frac{a_1 + a_2 + \ldots + a_{11}}{11}$ is equal to *(2010)*

3. Let $a_1, a_2, a_3 \ldots a_{100}$ be an arithmetic progression with $a_1 = 3$ and $S_p = \sum_{i=1}^{p} a_i, 1 \le p \le 100$. For any integer n with $1 \le n \le 20$, let $m = 5n$. If $\frac{S_m}{S_n}$ does not depend on n, then a_2 is *(2011)*

4. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then $k - 20 =$ *(JEE Adv. 2013)*

5. Let a, b, c be positive integers such that $\frac{b}{a}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is $b + 2$, then the value of $\frac{a^2 + a - 14}{a + 1}$ is *(JEE Adv. 2014)*

6. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6 : 11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is *(JEE Adv. 2015)*

7. The coefficient of x^9 in the expansion of $(1 + x)(1 + x^2)(1 + x^3) \ldots (1 + x^{100})$ is *(JEE Adv. 2015)*

Section-B JEE Main / AIEEE

1. If 1, $\log_9(3^{1-x}+2)$, $\log_3(4.3^x-1)$ are in A.P. then x equals **[2002]**
(a) $\log_3 4$ (b) $1-\log_3 4$
(c) $1-\log_4 3$ (d) $\log_4 3$

2. l, m, n are the p^{th}, q^{th} and r^{th} term of a G. P. all positive, then $\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix}$ equals **[2002]**
(a) –1 (b) 2 (c) 1 (d) 0

3. The value of $2^{1/4}.\,4^{1/8}.\,8^{1/16}\ldots\infty$ is **[2002]**
(a) 1 (b) 2 (c) 3/2 (d) 4

4. Fifth term of a GP is 2, then the product of its 9 terms is **[2002]**
(a) 256 (b) 512
(c) 1024 (d) none of these

5. Sum of infinite number of terms of GP is 20 and sum of their square is 100. The common ratio of GP is **[2002]**
(a) 5 (b) 3/5 (c) 8/5 (d) 1/5

6. $1^3-2^3+3^3-4^3+\ldots+9^3=$ **[2002]**
(a) 425 (b) –425 (c) 475 (d) –475

7. The sum of the series **[2003]**
$\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}$........... up to ∞ is equal to
(a) $\log_e\left(\frac{4}{e}\right)$ (b) $2\log_e 2$
(c) $\log_e 2-1$ (d) $\log_e 2$

8. If $S_n=\sum_{r=0}^{n}\frac{1}{{}^nC_r}$ and $t_n=\sum_{r=0}^{n}\frac{r}{{}^nC_r}$, then $\frac{t_n}{S_n}$ is equal to
(a) $\frac{2n-1}{2}$ (b) $\frac{1}{2}n-1$ **[2004]**
(c) $n-1$ (d) $\frac{1}{2}n$

9. Let T_r be the rth term of an A.P. whose first term is a and common difference is d. If for some positive integers $m, n,\ m\neq n,\ T_m=\frac{1}{n}$ and $T_n=\frac{1}{m}$, then $a-d$ equals **[2004]**
(a) $\frac{1}{m}+\frac{1}{n}$ (b) 1 (c) $\frac{1}{mn}$ (d) 0

10. The sum of the first n terms of the series $1^2+2.2^2+3^2+2.4^2+5^2+2.6^2+\ldots$ is $\frac{n(n+1)^2}{2}$ when n is even. When n is odd the sum is
(a) $\left[\frac{n(n+1)}{2}\right]^2$ (b) $\frac{n^2(n+1)}{2}$ **[2004]**
(c) $\frac{n(n+1)^2}{4}$ (d) $\frac{3n(n+1)}{2}$

11. The sum of series $\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots..$ is **[2004]**
(a) $\frac{(e^2-2)}{e}$ (b) $\frac{(e-1)^2}{2e}$
(c) $\frac{(e^2-1)}{2e}$ (d) $\frac{(e^2-1)}{2}$

12. If the coefficients of rth, $(r+1)$th, and $(r+2)$th terms in the binomial expansion of $(1+y)^m$ are in A.P., then m and r satisfy the equation **[2005]**
(a) $m^2-m(4r-1)+4r^2-2=0$
(b) $m^2-m(4r+1)+4r^2+2=0$
(c) $m^2-m(4r+1)+4r^2-2=0$
(d) $m^2-m(4r-1)+4r^2+2=0$

13. If $x=\sum_{n=0}^{\infty}a^n, y=\sum_{n=0}^{\infty}b^n, z=\sum_{n=0}^{\infty}c^n$ where a, b, c are in A.P and $|a|<1, |b|<1, |c|<1$ then x, y, z are in **[2005]**
(a) G.P.
(b) A.P.
(c) Arithmetic - Geometric Progression
(d) H.P.

14. The sum of the series **[2005]**
$1+\frac{1}{4.2!}+\frac{1}{16.4!}+\frac{1}{64.6!}+$ ....ad inf. is
(a) $\frac{e-1}{\sqrt{e}}$ (b) $\frac{e+1}{\sqrt{e}}$ (c) $\frac{e-1}{2\sqrt{e}}$ (d) $\frac{e+1}{2\sqrt{e}}$

15. Let a_1, a_2, a_3........... be terms on A.P. If $\frac{a_1+a_2+\ldots\ldots\ldots.a_p}{a_1+a_2+\ldots\ldots\ldots.+a_q}=\frac{p^2}{q^2}$, $p\neq q$, then $\frac{a_6}{a_{21}}$ equals **[2006]**
(a) $\frac{41}{11}$ (b) $\frac{7}{2}$ (c) $\frac{2}{7}$ (d) $\frac{11}{41}$

16. If $a_1, a_2, ..., a_n$ are in H.P., then the expression $a_1a_2 + a_2a_3 + ...+ a_{n-1}a_n$ is equal to **[2006]**

(a) $n(a_1 - a_n)$ (b) $(n-1)(a_1 - a_n)$

(c) na_1a_n (d) $(n-1)a_1a_n$

17. The sum of series $\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} -$ .... upto infinity is **[2007]**

(a) $e^{-\frac{1}{2}}$ (b) $e^{+\frac{1}{2}}$ (c) e^{-2} (d) e^{-1}

18. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of its progression is equals **[2007]**

(a) $\sqrt{5}$ (b) $\frac{1}{2}(\sqrt{5}-1)$

(c) $\frac{1}{2}(1-\sqrt{5})$ (d) $\frac{1}{2}\sqrt{5}$.

19. The first two terms of a geometric progression add up to 12. the sum of the third and the fourth terms is 48. If the terms of the geometric progression are alternately positive and negative, then the first term is **[2008]**

(a) –4 (b) –12 (c) 12 (d) 4

20. The sum to infinite term of the series

$1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...$ is **[2009]**

(a) 3 (b) 4 (c) 6 (d) 2

21. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n[th] minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference –2, then the time taken by him to count all notes is **[2010]**

(a) 34 minutes (b) 125 minutes

(c) 135 minutes (d) 24 minutes

22. A man saves ₹ 200 in each of the first three months of his service. In each of the subsequent months his saving increases by ₹ 40 more than the saving of immediately previous month. His total saving from the start of service will be ₹ 11040 after **[2011]**

(a) 19 months (b) 20 months

(c) 21 months (d) 18 months

23. **Statement-1:** The sum of the series 1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + ... + (361 + 380 + 400) is 8000.

Statement-2: $\sum_{k=1}^{n}\left(k^3 - (k-1)^3\right) = n^3$, for any natural number n. **[2012]**

(a) Statement-1 is false, Statement-2 is true.

(b) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.

(c) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.

(d) Statement-1 is true, statement-2 is false.

24. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777,....., is **[JEE M 2013]**

(a) $\frac{7}{81}(179 - 10^{-20})$ (b) $\frac{7}{9}(99 - 10^{-20})$

(c) $\frac{7}{81}(179 + 10^{-20})$ (d) $\frac{7}{9}(99 + 10^{-20})$

25. If $(10)^9 + 2(11)^1(10^8) + 3(11)^2(10)^7 + ...$ $+ 10(11)^9 = k(10)^9$, then k is equal to: **[JEE M 2014]**

(a) 100 (b) 110 (c) $\frac{121}{10}$ (d) $\frac{441}{100}$

26. Three positive numbers form an increasing G. P. If the middle term in this G.P. is doubled, the new numbers are in A.P. then the common ratio of the G.P. is: **[JEE M 2014]**

(a) $2-\sqrt{3}$ (b) $2+\sqrt{3}$ (c) $\sqrt{2}+\sqrt{3}$ (d) $3+\sqrt{2}$

27. The sum of first 9 terms of the series.

$\frac{1^3}{1} + \frac{1^3+2^3}{1+3} + \frac{1^3+2^3+3^3}{1+3+5} + ...$ **[JEE M 2015]**

(a) 142 (b) 192 (c) 71 (d) 96

28. If m is the A.M. of two distinct real numbers l and n(l, n > 1) and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals : **[JEE M 2015]**

(a) $4\,lmn^2$ (b) $4\,l^2m^2n^2$

(c) $4\,l^2mn$ (d) $4\,lm^2n$

29. If the 2[nd], 5[th] and 9[th] terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is : **[JEE M 2016]**

(a) 1 (b) $\frac{7}{4}$

(c) $\frac{8}{5}$ (d) $\frac{4}{3}$

30. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + ...,$ is $\frac{16}{5}$m, then m is equal to : **[JEE M 2016]**

(a) 100 (b) 99

(c) 102 (d) 101

6 Sequences and Series

Section-A : JEE Advanced/ IIT-JEE

A 1. 3050 2. 4 3. $\frac{n^2(n+1)}{2}$ 4. 4 : 1 or 1 : 4 5. $\frac{1}{4}(n+1)^2(2n-1)$

6. –3, 77

C	**1.** (c)	**2.** (b)	**3.** (c)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (d)
	8. (d)	**9.** (b)	**10.** (d)	**11.** (a)	**12.** (d)	**13.** (c)	**14.** (d)
	15. (c)	**16.** (c)	**17.** (c)	**18.** (d)	**19.** (b)		
D	**1.** (a, b, d)	**2.** (b, c)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (a, d)	**7.** (b, d)
	8. (a, d)	**9.** (a, d)					

E 1. 3 and 6 or 6 and 3 2. 9 3. yes, infinite 4. 5, 8, 12 7. $\frac{2^{mn}-1}{2^{mn}(2^n-1)}$ 8. $-\frac{1}{4}$

9. 3 11. $\frac{n(2n+1)(4n+1)-3}{3}$ 12. $\beta \in \left(-\infty, \frac{1}{3}\right], \gamma \in \left[-\frac{1}{27}, \infty\right)$

15. $G = (A_1 A_2 .....A_n H_1 H_2 ....H_n)^{1/2n}$. 18. 6

G	**1.** (b)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (a)	**6.** (b)	
H.	**1.** (c)						
I	**1.** 3	**2.** 0	**3.** 9	**4.** 5	**5.** 4	**6.** 9	**7.** 8

Section-B : JEE Main/ AIEEE

1. (b)	**2.** (d)	**3.** (b)	**4.** (b)	**5.** (b)	**6.** (a)	**7.** (a)	**8.** (d)
9. (d)	**10.** (b)	**11.** (b)	**12.** (c)	**13.** (d)	**14.** (d)	**15.** (d)	**16.** (d)
17. (d)	**18.** (b)	**19.** (b)	**20.** (a)	**21.** (a)	**22.** (c)	**23.** (b)	**24.** (c)
25. (a)	**26.** (b)	**27.** (d)	**28.** (d)	**29.** (d)	**30.** (d)		

Section-A JEE Advanced/ IIT-JEE

A. Fill in the Blanks

1. The sum of integers from 1 to 100 that are divisible by 2 or = sum of integers from 1 to 100 divisible by 2 + sum of integers from 1 to 100 divisible by 5 – sum of integers from 1 to 100 divisible by 10

$$=(2+4+6+...+100)+(5+10+15+...+100) -(10+20+...+100)$$

$$=\frac{50}{2}[2\times 2+49\times 2]+\frac{20}{2}[2\times 5+19\times 5]$$

$$=\frac{10}{2}[2\times 10+9\times 10]=2550+1050-550=3050$$

2. The given equation is

$$\log_7 \log_5 (\sqrt{x+5}+\sqrt{x})=0$$

$$\Rightarrow \log_5 (\sqrt{x+5}+\sqrt{x})=1$$

$$\Rightarrow \sqrt{x+5}+\sqrt{x}=5 \Rightarrow \sqrt{x+5}=5-\sqrt{x}$$

Squaring both sides

$$\Rightarrow x+5=25-10\sqrt{x}+x \Rightarrow 10\sqrt{x}=20$$

$$\Rightarrow \sqrt{x}=2 \Rightarrow x=4$$

3. When n is odd, let $n = 2m + 1$

$\therefore$ The req. sum

$$= 1^2+2.2^2+3^2+2.4^2+...+2\,(2m)^2+(2m+1)^2$$

$$= \Sigma(2m+1)^2+4[1^2+2^2+3^2+...+m^2]$$

$$= \frac{(2m+1)\,(2m+2)\,(4m+2+1)}{6}+\frac{4m\,(m+1)\,(2m+1)}{6}$$

$$= \frac{(2m+1)\,(m+1)}{6}\,[2\,(4m+3)+4m]$$

$$= \frac{(2m+1)\,(2m+2)\,(6m+3)}{6}=\frac{(2m+1)^2\,(2m+2)}{2}$$

$$= \frac{n^2(n+1)}{2} \quad [\because 2m+1=n)$$

4. Let a and b be two positive numbers.

Then, $\text{H.M.}=\frac{2ab}{a+b}$ and $G.M.=\sqrt{ab}$

ATQ $\quad HM : GM = 4:5$

$$\therefore \quad \frac{2ab}{(a+b)\sqrt{ab}}=\frac{4}{5}$$

$$\Rightarrow \frac{2\sqrt{ab}}{a+b}=\frac{4}{5} \Rightarrow \frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}}=\frac{5+4}{5-4}$$

$$\Rightarrow \frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2}=\frac{9}{1} \Rightarrow \frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}=3,-3$$

$$\Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}}=\frac{3+1}{3-1},\frac{-3+1}{-3-1} \Rightarrow \frac{\sqrt{a}}{\sqrt{b}}=2,\frac{1}{2} \Rightarrow \frac{a}{b}=4,\frac{1}{4}$$

$a : b = 4 : 1$ or $1 : 4$

5. Since n is an odd integer, $(-1)^{n-1}=1$ and $n-1, n-3, n-5$, ... are even integers.

We have

$n^3-(n-1)^3+(n-2)^3-(n-3)^3+.....+(-1)^{n-1}1^3$

$=n^3+(n-1)^3+(n-2)^3+......+1^3-2\,[(n-1)^3+(n-3)^3+.....+2^3]$

$=[n^3+(n-1)^3+(n-2)^3+......+1^3]$

$$-2\times 2^3\left[\left(\frac{n-1}{2}\right)^3+\left(\frac{n-3}{2}\right)^3+......+1^3\right]$$

[$\because$ n – 1, n – 3.........are even integers]

Here the first square bracket contain the sum of cubes of 1st n natural numbers. Whereas the second square bracket contains the sum of the cubes of natural numbers from 1 to $\left(\frac{n-1}{2}\right)$, where $n-1, n-3$, ... are even integers. Using the formula for sum of cubes of 1st n natural numbers we get the summation

$$=\left[\frac{n(n+1)}{2}\right]^2-16\left[\left\{\frac{1}{2}\left(\frac{n-1}{2}\right)\left(\frac{n-1}{2}+1\right)\right\}\right]^2$$

$$=\frac{1}{4}n^2(n+1)^2-16\,\frac{(n-1)^2(n+1)^2}{16\times 4}$$

$$=\frac{1}{4}(n+1)^2[n^2-(n-1)^2]=\frac{1}{4}(n+1)^2(2n-1)$$

6. It is given

$p+q=2, \quad pq=A$

and $\quad r+s=18, \quad rs=B$

and it is given that $p,q,r,$ are in A.P.

Therefore, let $p=a-3s, q=a-d, r=a+d$ and $s=a+3d$.

As $\quad p<q<r<s$, we have $d>0$

Now, $\quad 2=p+q=a-3d+a-d=2a-4d$

$\Rightarrow a-2d=1$ ....(1)

Again $18=r+s=a+d+a+3d$

$\Rightarrow 18=2a+4d \Rightarrow 9=a+2d.$ ....(2)

Subtracting (1) from (2)

$\Rightarrow 8=4d$

$\Rightarrow 2=d$ Putting in (2) we obtain a = 5

$\therefore \quad p=a-3d=5-6=-1, \quad q=a-d=5-2=3$

$r=a+d=5+2=7, \quad s=a+3d=5+6=11$

Therefore, $A=pq=-3$ and $B=rs=77$.

C. MCQs with ONE Correct Answer

1. **(c)** $\because x, y, z$ are the p^{th}, q^{th} and r^{th} terms of an AP.

$\therefore x=A+(p-1)D; y=A+(q-1)D;$

$z=A+(r-1)D$

$\Rightarrow x-y=(p-q)D;\ y-z=(q-r)D$

$z-x=(r-p)D$ ....(1)

where A is the first term and D is the common difference.

Also x, y, z are the p^{th}, q^{th}, and r^{th} terms of a GP.

$\therefore x=AR^{p-1}, y=AR^{q-1}, z=AR^{r-1}$

$\therefore x^{y-z}y^{z-x}z^{x-y}=(AR^{p-1})^{y-z}(AR^{q-1})^{z-x}(AR^{r-1})^{x-y}$

$=A^{y-z+z-x+x-y}R^{(p-1)(y-z)+(q-1)(z-x)+(r-1)(x-y)}$

$=A^0R^{(p-1)(q-r)D+(q-1)(r-p)D+(r-1)(p-q)D}=A^0R^0=1$

2. **(b)** $ar^2=4$ ....(1)

$a.\,ar.\,ar^2.ar^3.ar^4=a^5r^{10}=(ar^2)^5=4^5.$

3. **(c)** $2.\overline{357}=2+.357+0.000357+...\infty$

$$=2+\frac{357}{10^3}+\frac{357}{10^6}+....\infty=2+\frac{\frac{357}{10^3}}{1-\frac{1}{10^3}}=2+\frac{357}{999}=\frac{2355}{999}$$

4. **(a)** a, b, c are in G.P.

$b^2=ac$ ....(1)

$ax^2+2bx+c=0$

and $dx^2+2ex+f=0$ have a common root

Let it be α, then $a\alpha^2+2b\alpha+c=0$

$d\alpha^2+2e\alpha+f=0$

$$\Rightarrow \frac{\alpha^2}{2(bf-ec)}=\frac{\alpha}{cd-af}=\frac{1}{2(ae-bd)}$$

$$\Rightarrow \alpha^2=\frac{bf-ce}{ae-bd};\ \alpha=\frac{cd-af}{2\,(ae-bd)}$$

Substituting the value of α, we get

$$\frac{(cd-af)^2}{4\,(ae-bd)^2}=\frac{bf-ce}{ae-bd}$$

$\Rightarrow \quad (cd-af)^2=4\,(ae-bd)\,(bf-ce)$

Dividing both sides by a^2c^2 we get

$$\left(\frac{d}{a}-\frac{f}{c}\right)^2=4\left(\frac{e}{c}-\frac{bd}{ac}\right)\left(\frac{bf}{ac}-\frac{e}{a}\right)$$

$$\left(\frac{d}{a}-\frac{f}{c}\right)^2=4\left(\frac{e}{c}-\frac{d}{b}\right)\left(\frac{f}{b}-\frac{e}{a}\right) \quad \text{[Using eq. (1)]}$$

$$\Rightarrow \frac{d^2}{a^2}+\frac{f^2}{c^2}-\frac{2df}{ac}=\frac{4ef}{cb}-\frac{4e^2}{ac}-\frac{4df}{b^2}+\frac{4de}{ab}$$

$$\Rightarrow \frac{d^2}{a^2}+\frac{f^2}{c^2}+\frac{4e^2}{b^2}+2\frac{d}{a}.\frac{f}{c}-4\frac{e}{b}.\frac{f}{c}-4.\frac{d}{a}.\frac{e}{b}=0$$

[Using eq.(1)]

$$\Rightarrow \left(\frac{d}{a}+\frac{f}{c}-2\frac{e}{b}\right)^2=0 \Rightarrow \frac{d}{a}+\frac{f}{c}=\frac{2e}{b}$$

$\Rightarrow \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

5. **(c)** Let $S=\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+....n\ terms$

$=\left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)....\text{n terms}$

$=(1+1+1+....n\text{ terms})-\left(\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}+....+\frac{1}{2^n}\right)$

$=n-\left[\frac{\frac{1}{2}\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}}\right]=n-1+2^{-n}$

6. **(c)** We know that $\log_2 4=2$ and $\log_2 8=3$

$\therefore$ $\log_2 7$ lies between 2 and 3

$\therefore$ $\log_2 7$ is either rational or irrational but not integer or prime number.

If possible let $\log_2 7=\frac{p}{q}$ (a rational number)

$\Rightarrow$ $2^{p/q}=7\Rightarrow 2^p=7^q$

$\Rightarrow$ even number = odd number

$\therefore$ We get a contradiction, so assumption is wrong.

Hence $\log_2 7$ must be an irrational number.

7. **(d)** In $(a+c)$, In $(a-c)$, In $(a-2b+c)$ are in A.P.

$\Rightarrow$ $a+c, a-c, a-2b+c$ are in G.P.

$\Rightarrow$ $(c-a)^2=(a+c)(a-2b+c)$

$\Rightarrow$ $(c-a)^2=(a+c)^2-2b(a+c)$

$\Rightarrow$ $2b(a+c)=(a+c)^2-(c-a)^2$

$\Rightarrow$ $2b(a+c)=4ac\Rightarrow b=\frac{2ac}{a+c}$

$\Rightarrow$ a, b, c are in H.P.

8. **(d)** $a_1=h_1=2, a_{10}=h_{10}=3$

$3=a_{10}=2+9d\ \Rightarrow\ d=1/9$

$\therefore$ $a_4=2+3d=7/3$

$3=h_{10}\Rightarrow\frac{1}{3}=\frac{1}{h_{10}}=\frac{1}{2}+9D\quad\therefore\ D=-\frac{1}{54}$

$\frac{1}{h_7}=\frac{1}{2}+6D=\frac{1}{2}-\frac{1}{9}=\frac{7}{18}\qquad\therefore\ a_4h_7=\frac{7}{3}\times\frac{18}{7}=6.$

9. **(b)** $\frac{1}{H}=\frac{1}{2}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)=\frac{\alpha+\beta}{2\alpha\beta}=\frac{(4+\sqrt{5})}{(5+\sqrt{2})}\cdot\frac{(5+\sqrt{2})}{(8+2\sqrt{5})}\cdot\frac{1}{2}=\frac{1}{4}$

$\therefore$ $H=4.$

10. **(d)** Sum = 4 and second term = 3/4, it is given that first term is a and common ratio r

$\Rightarrow$ $\frac{a}{1-r}=4$ and $ar=3/4\Rightarrow r=\frac{3}{4a}$

Therefore, $\frac{a}{1-\frac{3}{4a}}=4\Rightarrow\frac{4a^2}{4a-3}=4$

or $a^2-4a+3=0\Rightarrow(a-1)(a-3)=0$

$\Rightarrow$ $a=1$ or 3

When $a=1, r=3/4$ and when $a=3, r=1/4$

11. **(a)** α, β are the roots of $x^2-x+p=0$

$\therefore$ $\alpha+\beta=1$ ....(1)

$\alpha\beta=p$ ...(2)

γ, δ are the roots of $x^2-4x+q=0$

$\therefore$ $\gamma+\delta=4$ ....(3)

$\gamma\delta=q$ ....(4)

$\alpha, \beta, \gamma, \delta$ are in G.P.

$\therefore$ Let $\alpha=a;\ \beta=ar,\ \gamma=ar^2,\ \delta=ar^3.$

Substituting these values in equations (1), (2), (3) and (4), we get

$a+ar=1$ ....(5)

$a^2r=p$ ....(6)

$ar^2+ar^3=4$ ....(7)

$a^2r^5=q$ ....(8)

Dividing (7) by (5) we get

$\frac{ar^2(1+r)}{a(1+r)}=\frac{4}{1}\Rightarrow r^2=4\Rightarrow r=2,-2$

(5) $\Rightarrow a=\frac{1}{1+r}=\frac{1}{1+2}$ or $\frac{1}{1-2}=\frac{1}{3}$ or -1

As p is an integer (given), r is also an integer (2 or –2).

$\therefore$ (6) $\Rightarrow a\neq\frac{1}{3}$. Hence $a=-1$ and $r=-2.$

$\therefore$ $p=(-1)^2\times(-2)=-2$

$q=(-1)^2\times(-2)^5=-32$

12. **(d)** a, b, c, d are in A.P.

$\therefore$ d, c, b, a are also in A.P.

$\Rightarrow$ $\frac{d}{abcd}, \frac{c}{abcd}, \frac{b}{abcd}, \frac{a}{abcd}$ are also in A.P.

$\Rightarrow$ $\frac{1}{abc}, \frac{1}{abd}, \frac{1}{acd}, \frac{1}{bcd}$ are in A.P.

$\Rightarrow$ abc, abd, acd, bcd are in H.P.

13. **(c)** ATQ $2+5+8+....2n$ terms $=57+59+61+....n$ terms

$\Rightarrow$ $\frac{2n}{2}[4+(2n-1)3]=\frac{n}{2}[114+(n-1)2]$

$\Rightarrow$ $6n+1=n+56\Rightarrow 5n=55\Rightarrow n=11$

14. **(d)** Given that a, b, c are in A.P.

$\Rightarrow$ $2b=a+c$

$\Rightarrow$ but given $a+b+c=3/2\ \Rightarrow\ 3b=3/2$

$\Rightarrow$ $b=1/2$ and then $a+c=1$

Again a^2, b^2, c^2, are in G.P. $\Rightarrow b^4=a^2c^2$

$\Rightarrow$ $b^2=\pm ac\Rightarrow ac=\frac{1}{4}$ or $-\frac{1}{4}$

and $a+c=1$ ....(1)

Considering $a+c=1$ and $ac=1/4$

$\Rightarrow$ $(a-c)^2=1-1=0\ \Rightarrow a=c$ but

$a\neq c$ as given that $a<b<c$

$\therefore$ We consider $a+c=1$ and $ac=-1/4$

$\Rightarrow$ $(a-c)^2=1+1=2\Rightarrow a-c=\pm\sqrt{2}$

but $a<c\Rightarrow a-c=-\sqrt{2}$ ....(2)

Solving (1) and (2) we get $a=\frac{1}{2}-\frac{1}{\sqrt{2}}$

15. (c) $\frac{x}{1-r}=5 \Rightarrow r=1-\frac{x}{5}$

Since G.P. contains infinite terms

$\therefore \quad -1<r<1$

$\Rightarrow \quad -1<1-\frac{x}{5}<1 \Rightarrow -2<-\frac{x}{5}<0$

$\Rightarrow \quad -10<x<0. \Rightarrow 0<\frac{x}{5}<2$

$\Rightarrow \quad 0<x<10.$

16. (c) In the quadratic equation $ax^2+bx+c=0$

$\Delta=b^2-4ac$ and $\alpha+\beta=-\frac{b}{a}, \alpha\beta=\frac{c}{a}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\frac{b^2}{a^2}-\frac{2c}{a}=\frac{b^2-2ac}{a^2}$

and $\alpha^3+\beta^3=-\frac{b^3}{a^3}-\frac{3c}{a}\left(-\frac{b}{a}\right)=-\left(\frac{b^3-3abc}{a^3}\right)$

Given $\alpha+\beta, \alpha^2+\beta^2, \alpha^3+\beta^3$ are in G.P.

$\Rightarrow -\frac{b}{a}, \frac{b^2-2ac}{a^2}, -\frac{(b^3-3\,abc)}{a^3}$ are in G.P.

$\Rightarrow \left(\frac{b^2-2\,ac}{a^2}\right)^2=\frac{b}{a}\left(\frac{b^3-3\,abc}{a^3}\right)$

$\Rightarrow b^4+4a^2c^2-4ab^2c=b^4-3ab^2c$

$\Rightarrow 4a^2c^2-ab^2c=0 \Rightarrow ac\,\Delta=0$

$\Rightarrow c\,\Delta=0$ ($\because$ In quadratic $a \neq 0$)

17. (c) Given that for an A.P, $S_n=cn^2$

Then $T_n=S_n-S_{n-1}=cn^2-c(n-1)^2$

$=(2n-1)c$

$\therefore$ Sum of squares of n terms of this A.P

$=\sum T_n^2 == \sum(2n-1)^2.c^2$

$=c^2\left[4\sum n^2-4\sum n+n\right]$

$=c^2\left[\frac{4n(n+1)(2n+1)}{6}-\frac{4n(n+1)}{2}+n\right]$

$=c^2n\left[\frac{2(2n^2+3n+1)-6(n+1)+3}{3}\right]$

$=c^2n\left[\frac{4n^2-1}{3}\right]=\frac{n(4n^2-1)c^2}{3}$

18. (d) $\because a_1, a_2, a_3, ...$ are in H.P.

$\therefore \frac{1}{a_1}, \frac{1}{a_2}, \frac{1}{a_3}$ .... are in A.P.

$\therefore \quad \frac{1}{a_1}=\frac{1}{5}$ and $\frac{1}{a_{20}}=\frac{1}{25}$

$\frac{1}{a_1}+19d=\frac{1}{a_{20}} \Rightarrow \frac{1}{5}+19d=\frac{1}{25} \Rightarrow d=\frac{-4}{475}$

Now $\frac{1}{a_n}=\frac{1}{5}+(n-1)\left(\frac{-4}{475}\right)$

Clearly $a_n<0$ if $\frac{1}{a_n}<0 \Rightarrow \frac{1}{5}-\frac{4n}{475}+\frac{4}{475}<0$

$\Rightarrow -4n<-99$ or $n>\frac{99}{4}=24\frac{3}{4} \quad \therefore n \geq 25$

Hence least value of n is 25.

19. (b) $\log_c b_1, \log_c b_2, ----, \log_c b_{101}$ are in A.P.

$\Rightarrow \quad b_1, b_2, ---- b_{101}$ are in G.P.

Also $a_1, a_2, ---- a_{101}$ are in A.P.

where $a_1=b_1$ are $a_{51}=b_{51}$.

$\therefore \quad b_2, b_3, ---, b_{50}$ and GM's and $a_2, a_3, ----, a_{50}$ are AM's between b_1 and b_{51}.

$\because$ GM < AM $\Rightarrow b_2<a_2, b_3<a_3, ----, b_{50}<a_{50}$

$\therefore \quad b_1+b_2+----+b_{51}<a_1+a_2+----+a_{51}$

$\Rightarrow \quad t<s$

Also a_1, a_{51}, a_{101} are in AP

b_1, b_{51}, b_{101} are in GP

$\because \quad a_1=b_1$ and $a_{51}=b_{51}$

$\therefore \quad b_{101}>a_{101}$

D. MCQs with ONE or MORE THAN ONE Correct

1. (a,b,d) Let x be the first term and y the $(2n-1)$th terms of AP, GP and HP whose nth terms are a, b, c respectively.

For AP, $y=x+(2n-2)\,d$

$\Rightarrow \quad d=\frac{y-x}{2(n-1)}$

$\therefore \quad a=x+(n-1)\,d=x+\frac{1}{2}(y-x)=\frac{1}{2}(x+y)$ ....(1)

For G.P. $\quad y=xr^{2n-2} \quad \Rightarrow r=\left(\frac{y}{x}\right)^{\frac{1}{2n-2}}$

$\therefore \quad b=xr^{n-1}=x.\left(\frac{y}{x}\right)^{1/2}=\sqrt{xy}$ ....(2)

For H.P. $\quad \frac{1}{y}=\frac{1}{x}+(2n-2)d_1$

$\Rightarrow \quad d_1=\frac{x-y}{2xy\,(n-1)}$

$\therefore \quad \frac{1}{c}=\frac{1}{x}+(n-1)\,d_1=\frac{1}{x}+\frac{x-y}{2xy}$

$\frac{1}{c}=\frac{x+y}{2xy} \Rightarrow c=\frac{2xy}{x+y}$ ....(3)

Thus from (1), (2) and (3), a, b, c are A.M., G.M. and H.M. respectively of x and y.

2. (b, c) We have for $0<\phi<\frac{\pi}{2}$

$x=\sum_{n=0}^{\infty}\cos^{2n}\phi=1+\cos^2\phi+\cos^4\phi+....\infty$

$\frac{1}{1-\cos^2\phi}=\frac{1}{\sin^2\phi}$ ....(1)

[Using sum of infinite G.P. $\cos^2\alpha$ being < 1]

$$y=\sum_{n=0}^{\infty}\sin^{2n}\phi=1+\sin^2\phi+\sin^4\phi+....\infty$$

$$=\frac{1}{1-\sin^2\phi}=\frac{1}{\cos^2\phi} \quad ....(2)$$

$$z=\sum_{n=0}^{\infty}\cos^{2n}\phi\sin^{2n}\phi$$

$$=1+\cos^2\phi\sin^2\phi+\cos^4\phi\sin^4\phi+....\infty$$

$$=\frac{1}{1-\cos^2\phi\sin^2\phi} \quad ....(3)$$

Substituting the values of $\cos^2\phi$ and $\sin^2\phi$ in (3), from (1) and (2), we get

$$z=\frac{1}{1-\frac{1}{x}.\frac{1}{y}}\Rightarrow z=\frac{xy}{xy-1}\Rightarrow xyz-z=xy\Rightarrow xyz=xy+z.$$

Also, $x+y+z=\dfrac{1}{\cos^2\phi}+\dfrac{1}{\sin^2\phi}+\dfrac{1}{1-\cos^2\phi\sin^2\phi}$

$$\frac{[\sin^2\phi(1-\cos^2\phi\sin^2\phi)+\cos^2\phi(1-\cos^2\phi\sin^2\phi)+\cos^2\phi\sin^2\phi]}{\cos^2\phi\sin^2\phi\,(1-\cos^2\phi\sin^2\phi)}$$

$$=\frac{(\sin^2\phi+\cos^2\phi)\,(1-\cos^2\phi\sin^2\phi)+\cos^2\phi\sin^2\phi}{\cos^2\phi\sin^2\phi\,(1-\cos^2\phi\sin^2\phi)}$$

$$=\frac{1}{\cos^2\phi\sin^2\phi\,(1-\cos^2\phi\sin^2\phi)}=xyz$$

Thus (b) and (c) both are correct.

3. **(b)** Putting $\theta=0$, we get $b_0=0$

$\therefore$ $\sin\, n\theta=\sum_{r=1}^{n}b_r\sin^r\theta \Rightarrow \dfrac{\sin n\theta}{\sin\theta}=\sum_{r=1}^{n}b_r(\sin\theta)^{r-1}$

$=b_1+b_2\sin\theta+b_3\sin^2\theta+......+b_n\sin^{n-1}\theta$

Taking limit as $\theta\to 0$, we obtain

$$\lim_{\theta\to 0}\frac{\sin n\theta}{\sin\theta}=b_1\Rightarrow b_1=n.$$

4. **(c)** $T_m=a+(m-1)\,d=1/n$ and $t_n=a+(n-1)d=1/m$

$\Rightarrow (m-n)\,d=1/n-1/m=(m-n)/mn \Rightarrow d=1/mn$

$\Rightarrow a=\dfrac{1}{mn}$

$\therefore$ $t_{mn}=a+(mn-1)\,d=\dfrac{1}{mn}+(mn-1)\dfrac{1}{mn}$

$=\dfrac{1}{mn}+1-\dfrac{1}{mn}=1$

5. **(b)** If x, y, z are in G.P. ($x,\ y,\ z>1$); $\log x$, $\log y$, $\log z$ will be in A.P.

$\Rightarrow$ $1+\log x$, $1+\log y$, $1+\log z$ will also be in A.P.

$\Rightarrow$ $\dfrac{1}{1+\log x},\dfrac{1}{1+\log y},\dfrac{1}{1+\log z}$ will be in H.P.

6. **(a, d)** We have

$$a(n)=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+.....+\frac{1}{2^n-1}$$

$$=1+\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right)+\left(\frac{1}{8}+.....+\frac{1}{15}\right)+...+\left(\frac{1}{2^{n-1}}+.....+\frac{1}{2^n-1}\right)$$

$$<1+\left(\frac{1}{2}+\frac{1}{2}\right)+\left(\frac{1}{4}+\frac{1}{4}+.....+\frac{1}{4}\right)+\left(\frac{1}{8}+\frac{1}{8}+...+\frac{1}{8}\right)+..+\left(\frac{1}{2^{n-1}}+..+\frac{1}{2^{n-1}}\right)$$

$$<1+\frac{2}{2}+\frac{4}{4}+\frac{8}{8}+.....+\frac{2^{n-1}}{2^{n-1}}=1+1+.....+1=n$$

Thus, $a\,(100)<100$

Also

$$\text{a (n)}=1+\frac{1}{2}+\left(\frac{1}{3}+\frac{1}{4}\right)+\left(\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}\right)+.....+\left(\frac{1}{2^{n-1}+1}+..+\frac{1}{2^n}\right)-\frac{1}{2^n}$$

$$>1+\frac{1}{2}+\left(\frac{1}{4}+\frac{1}{4}\right)+\left(\frac{1}{8}+\frac{1}{8}+...+\frac{1}{8}\right)+...+\left(\frac{1}{2^n}+..+\frac{1}{2^n}\right)-\frac{1}{2^n}$$

$$=1+\frac{1}{2}+\frac{2}{4}+\frac{4}{8}+...+\frac{2^{n-1}}{2^n}-\frac{1}{2^n}$$

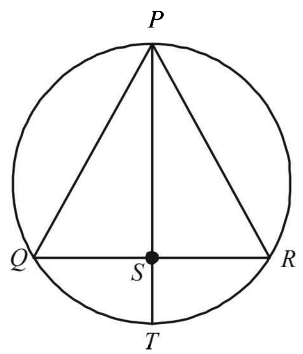
$$=1+\underbrace{\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+.....+\frac{1}{2}\right)}_{n\text{ times}}-\frac{1}{2^n}$$

$$=1+\frac{n}{2}-\frac{1}{2^n}=\left(1-\frac{1}{2^n}\right)+\frac{n}{2}$$

Thus, $a\,(200)>\left(1-\dfrac{1}{2^{200}}\right)+\dfrac{200}{2}>100,$

i.e. $a\,(200)>100.$

7. **(b, d)**



We know by geometry $PS\times ST=QS\times SR$ ...(1)

$\because$ S is not the centre of circulm circle,

$PS \neq ST$

And we know that for two unequal real numbers.
H.M. < G.M.

$$\Rightarrow \frac{2}{\frac{1}{PS}+\frac{1}{ST}} < \sqrt{PS \times ST} \Rightarrow \frac{1}{PS}+\frac{1}{ST} > \frac{2}{\sqrt{PS \times ST}}$$

$$\Rightarrow \frac{1}{PS}+\frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}} \text{ [using eqn (1)] ...(2)}$$

$\therefore$ (b) is the correct option.

Also $\sqrt{QS \times SR} < \frac{QS+SR}{2}$ ($\because$ GM < AM)

$$\Rightarrow \frac{1}{\sqrt{QS \times SR}} > \frac{2}{QR} \Rightarrow \frac{2}{\sqrt{QS \times SR}} > \frac{4}{QR} \text{ ...(3)}$$

From equations (2) and (3) we get $\frac{1}{PS}+\frac{1}{ST} > \frac{4}{QR}$

$\therefore$ (d) is also the correct option.

8. **(a,d)** We have $S_n = \sum_{k=1}^{n} \frac{n}{n^2+kn+k^2}$

and $T_n = \sum_{k=0}^{n-1} \frac{n}{n^2+kn+k^2}$ $n = 1, 2, 3.........$

For $n = 1$ we get

$$S_1 = \frac{1}{1+1+1} = \frac{1}{3} = 0.3 \text{ and } T_1 = \frac{1}{1+0} = 1$$

Also $\frac{\pi}{3\sqrt{3}} = \frac{\pi\sqrt{3}}{9} = \frac{3.14 \times 1.73}{9} = 0.34 \times 1.73 = 0.58$

$\therefore S_1 < \frac{\pi}{3\sqrt{3}} < T_1$, $\therefore S_n < \frac{\pi}{3\sqrt{3}}$ and $T_n > \frac{\pi}{3\sqrt{3}}$

9. **(a,d)** $S_n = -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + ...$

$= (3^2 + 7^2 + 11^2 + ...) + (4^2 + 8^2 + 12^2 + ...)$
$-(1^2 + 5^2 + 9^2 + ...) - (2^2 + 6^2 + 10^2 + ...)$

$$= \sum_{r=1}^{n}(4r-1)^2 + \sum_{r=1}^{n}(4r)^2 - \sum_{r=1}^{n}(4r-3)^2 - \sum_{r=1}^{n}(4r-2)^2$$

$$= \left[\sum_{r=1}^{n}(4r-1)^2 - (4r-3)^2\right] + 4\left[\sum_{r=1}^{n}(2r)^2 - (2r-1)^2\right]$$

$$= 8\sum_{r=1}^{n}(2r-1) + 4\sum_{r=1}^{n}(4r-1)$$

$$= 8\left[2\frac{n(n+1)}{2} - n\right] + 4\left[4\frac{n(n+1)}{2} - n\right]$$

$= 8n^2 + 8n^2 + 4n = 16n^2 + 4n$

For $n = 8$, $16n^2 + 4n = 1056$

and for $n = 9$, $16n^2 + 4n = 1332$

E. Subjective Problems

1. Let the two numbers be a and b, then

$$\frac{2ab}{a+b} = 4 ...(1); \frac{a+b}{2} = A; \sqrt{ab} = G$$

Also $2A + G^2 = 27 \Rightarrow a + b + ab = 27$ ....(2)

Putting $ab = 27 - (a+b)$ in eqn. (1), we get

$$\frac{54-2(a+b)}{a+b} = 4 \Rightarrow a+b = 9 \text{ then } ab = 27 - 9 = 18$$

Solving the two we get $a = 6, b = 3$ or $a = 3, b = 6$, which are the required numbers.

2. Let there be n sides in the polygon.

Then by geometry, sum of all n interior angles of polygon $= (n - 2) \times 180°$

Also the angles are in A.P. with the smallest angle = 120°, common difference = 5°

$\therefore$ Sum of all interior angles of polygon

$$= \frac{n}{2}[2 \times 120 + (n-1) \times 5]$$

Thus we should have

$$\frac{n}{2}[2 \times 120 + (n-1) \times 5] = (n-2) \times 180$$

$$\Rightarrow \frac{n}{2}[5n + 235] = (n-2) \times 180$$

$$\Rightarrow 5n^2 + 235n = 360n - 720$$

$$\Rightarrow 5n^2 - 125n + 720 = 0 \Rightarrow n^2 - 25n + 144 = 0$$

$$\Rightarrow (n-16)(n-9) = 0 \Rightarrow n = 16, 9$$

Also if $n = 16$ then 16th angle $= 120 + 15 \times 5 = 195° > 180°$

$\therefore$ not possible. Hence $n = 9$.

3. If possible let for a G.P.

$T_p = 27 = AR^{p-1}$ ....(1)

$T_q = 8 = AR^{q-1}$ ....(2)

$T_r = 12 = AR^{r-1}$ ....(3)

From (1) and (2)

$$R^{p-q} = \frac{27}{8} \Rightarrow R^{p-q} = (3/2)^3 \text{ ....(4)}$$

From (2) and (3):

$$R^{q-r} = \frac{8}{12} \Rightarrow R^{q-r} = (3/2)^{-1} \text{ ....(5)}$$

From (4) and (5):

$R = 3/2; p - q = 3; q - r = -1$

$p - 2q + r = 4;$ $p, q, r \in N$ ....(6)

As there can be infinite natural numbers for p, q and r to satisfy equation (6)

$\therefore$ There can be infinite G.P.'s.

4. $2 < a, b, c < 18$ $a + b + c = 25$ ....(1)

2, a, b are in AP $\Rightarrow 2a = b + 2$

$\Rightarrow 2a - b = 2$ ....(2)

b, c, 18 are in GP $\Rightarrow c^2 = 18b$ ....(3)

From (2) $\Rightarrow a = \frac{b+2}{2}$

(1) $\Rightarrow \frac{b+2}{2} + b + c = 25 \Rightarrow 3b = 48 - 2c$

(3) $\Rightarrow c^2 = 6 \ (48 - 2c) \Rightarrow c^2 + 12c - 288 = 0$

$\Rightarrow c = 12, -24$ (rejected) $\Rightarrow a = 5, b = 8, c = 12$

5. Given that $a, b, c > 0$

We know for +ve numbers A.M. ≥ G.M.

$\therefore$ For +ve numbers a, b, c we get

$$\frac{a+b+c}{3} \geq \sqrt[3]{abc} \text{ ....(1)}$$

Also for +ve numbers $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$, we get

$$\frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3} \geq \sqrt[3]{\frac{1}{abc}} \quad ....(2)$$

Multiplying in eqs (1) and (2) we get

$$\left(\frac{a+b+c}{3}\right)\left(\frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3}\right) \geq \sqrt[3]{abc} \times \frac{1}{\sqrt[3]{abc}}$$

$$\Rightarrow \quad (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9 \qquad \text{Proved.}$$

6. Given that $n = p_1^{\alpha_1}.\, p_2^{\alpha_2}.\, p_3^{\alpha_3} .... p_k^{\alpha_k}$ ....(1)

Where $n \in N$ and $p_1, p_2, p_3, ...p_k$ are distinct prime numbers.

Taking log on both sides of eq. (1), we get

$\log n = \alpha_1 \log p_1 + \alpha_2 \log p_2 + ... + \alpha_k \log p_k$ ....(2)

Since every prime number is such that

$p_i \geq 2$

$\therefore \quad \log_e p_i \geq \log_e 2$ ....(3)

$\forall \quad i = 1\,(1)\,k$

Using (2) and (3) we get

$\log n \geq \alpha_1 \log 2 + \alpha_2 \log 2 + \alpha_3 \log 2 + ... + \alpha_k \log 2$

$\Rightarrow \quad \log n \geq (\alpha_1 + \alpha_2 + ... + \alpha_k) \log 2$

$\Rightarrow \quad \log n \geq k \log 2$ **Proved.**

7. The given series is

$$\sum_{r=0}^{n} (-1)^r\, {}^nC_r \left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+.... \text{ up to } m \text{ terms}\right]$$

$$\sum_{r=0}^{n} (-1)^r\, {}^nC_r \left[\left(\frac{1}{2}\right)^r+\left(\frac{3}{4}\right)^r+\left(\frac{7}{8}\right)^r+\left(\frac{15}{16}\right)^r+.... \text{to } m \text{ terms}\right]$$

Now, $\sum_{r=0}^{n} (-1)^r\, {}^nC_r \left(\frac{1}{2}\right)^r = 1 - {}^nC_1.\frac{1}{2} + {}^nC_2.\frac{1}{2^2} - {}^nC_3.\frac{1}{2^3} + ...$

$$= \left(1-\frac{1}{2}\right)^n = \frac{1}{2^n}$$

Similarly, $\sum_{r=0}^{n} (-1)^r\, {}^nC_r \left(\frac{3}{4}\right)^r = \left(1-\frac{3}{4}\right)^n = \frac{1}{4^n}$ etc.

Hence the given series is,

$$= \frac{1}{2^n}+\frac{1}{4^n}+\frac{1}{8^n}+\frac{1}{16^n}+.... \text{to } m \text{ terms}$$

$$= \frac{\frac{1}{2^n}\left(1-\left(\frac{1}{2^n}\right)^m\right)}{1-\frac{1}{2^n}} \qquad \text{[Summing the G.P.]}$$

$$= \frac{2^{mn}-1}{2^{mn}(2^n-1)}$$

8. The given equation is

$\log_{(2x+3)}(6x^2+23+21)$

$= 4 - \log_{3x+7}(4x^2+12x+9)$

$\Rightarrow \quad \log_{(2x+3)}(6x^2+23x+21)$

$+ \log_{(3x+7)}(4x^2+12x+9) = 4$

$\Rightarrow \quad \log_{(2x+3)}(2x+3)(3x+7) + \log_{(3x+7)}(2x+3)^2\, x = 4$

$\Rightarrow \quad 1 + \log_{(2x+3)}(3x+7) + 2\log_{(3x+7)}(2x+3) = 4$

$$\Rightarrow \quad \log_{(2x+3)}(3x+7) + \frac{2}{\log_{(2x+3)}(3x+7)} = 3$$

Let $\log_{(2x+3)}(3x+7) = y$ ....(1)

$$\Rightarrow \quad y + \frac{2}{y} = 3 \Rightarrow y^2 - 3y + 2 = 0$$

$\Rightarrow \quad (y-1)\,(y-2) = 0 \Rightarrow y = 1, 2$

Substituting the values of y in (1), we get

$\Rightarrow \quad \log_{(2x+3)}(3x+7) = 1$ and $\log_{(2x+3)}(3x+7) = 2$

$\Rightarrow \quad 3x+7 = 2x+3$ and $3x+7 = (2x+3)^2$

$\Rightarrow \quad x = -4$ and $4x^2+9x+2 = 0$

$\Rightarrow \quad x = -4$ and $(x+2)\,(4x+1) = 0$

$\Rightarrow \quad x = -4$ and $x = -2, x = -\frac{1}{4}$

As $\log_a x$ is defined for $x > 0$ and $a > 0$ $(a \neq 1)$, the possible value of x should satisfy all of the following inequalities :

$\Rightarrow \quad 2x+3 > 0$ and $3x+7 > 0$

Also $2x+3 \neq 1$ and $3x+7 \neq 1$

Out of $x = -4, x = -2$ and $x = -\frac{1}{4}$ only $x = -\frac{1}{4}$ satisfies the above inequalities.

So only solution is $x = -\frac{1}{4}$.

9. Given that $\log_3 2$, $\log_3(2^x - 5)$, $\log_3(2^x - 7/2)$ are in A.P.

$\Rightarrow \quad 2\log_3(2^x-5) = \log_3 2 + \log_3(2^x - 7/2)$

$$\Rightarrow \quad (2^x-5)^2 = 2\left(2^x - \frac{7}{2}\right)$$

$\Rightarrow \quad (2^x)^2 - 10.2^x + 25 - 2.2^x + 7 = 0$

$\Rightarrow \quad (2^x)^2 - 12.2^x + 32 = 0$

Let $2^x = y$, then we get,

$y^2 - 12y + 32 = 0 \Rightarrow (y-4)\,(y-8) = 0$

$\Rightarrow \quad y = 4$ or $8 \Rightarrow 2^x = 2^2$ or $2^3 \Rightarrow x = 2$ or 3

But for $\log_3(2^x-5)$ and $\log_3(2^x-7/2)$ to be defined

$2^x - 5 > 0$ and $2^x - 7/2 > 0$

$\Rightarrow \quad 2^x > 5$ and $2^x > 7/2$

$\Rightarrow \quad 2^x > 5$

$\Rightarrow \quad x \neq 2$ and therefore $x = 3$.

10. Let a and b be two numbers and $A_1, A_2, A_3, ... A_n$ be n A.M's between a and b.

Then a, $A_1, A_2, ... A_n$, b are in A.P.

There are $(n+2)$ terms in the series, so that

$$a + (n+1)\,d = b \Rightarrow d = \frac{b-a}{n+1}$$

$$\therefore \quad A_1 = a + \frac{b-a}{n+1} = \frac{an+b}{n+1}$$

$$\therefore \quad p = \frac{an+b}{n+1} \quad ....(1)$$

The first H.M. between a and b, when nHM's are inserted between a and b can be obtained by replacing a by $\frac{1}{a}$ and b by $\frac{1}{b}$ in eq. (1) and then taking its reciptocal.

Therefore, $q = \frac{1}{\frac{\left(\frac{1}{a}\right)n + \frac{1}{b}}{n+1}} = \frac{(n+1)ab}{bn+a}$

$\therefore \quad q = \frac{(n+1)ab}{a+bn}$ ....(2)

We have to prove that q cannot lie between p and $\frac{(n+1)^2}{(n-1)^2}p.$

Now, $n+1 > n-1 \Rightarrow \frac{n+1}{n-1} > 1$

$\Rightarrow \left(\frac{n+1}{n-1}\right)^2 > 1$ or $p\left(\frac{n+1}{n-1}\right)^2 > p$

$\Rightarrow \quad p < p\left(\frac{n+1}{n-1}\right)^2$ ....(3)

Now to prove the given, we have to show that q is less than p.

For this, let, $\frac{p}{q} = \frac{(na+b)\,(nb+a)}{(n+1)^2 ab}$

$\Rightarrow \quad \frac{p}{q} - 1 = \frac{n(a^2+b^2) + ab(n^2+1) - (n+1)^2 ab}{(n+1)^2 ab}$

$\Rightarrow \quad \frac{p}{q} - 1 = \frac{n(a^2+b^2-2ab)}{(n+1)^2 ab}$

$\Rightarrow \quad \frac{p}{q} - 1 = \frac{n}{(n+1)^2}\left(\frac{a-b}{\sqrt{ab}}\right)^2 = \frac{n}{(n+1)^2}\left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right)^2$

$\Rightarrow \quad \frac{p}{q} - 1 > 0$

$\Rightarrow$ (provided a and b and hence p and q are +ve)
$p > q$ ....(4)

From 3 and (4), we get, $q < p < \left(\frac{n+1}{n+1}\right)^2 p$

$\therefore$ q can not lie between p and $\left(\frac{n+1}{n+1}\right)^2 p$, if a and b are +ve numbers.

11. We have,

$$S_1 = 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + ...\infty$$

$$S_2 = 2 + 2.\frac{1}{3} + 2\left(\frac{1}{3}\right)^2 + ...\infty$$

....

....

$$S_3 = 3 + 3.\frac{1}{4} + 3\left(\frac{1}{4}\right)^2 + ...\infty$$

$$S_n = n + n.\frac{1}{n+1} + n\left(\frac{1}{n+1}\right)^2 + ...\infty$$

$\Rightarrow \quad S_1 = \frac{1}{1-\frac{1}{2}} = 2 \qquad \left[Using\ S_\infty = \frac{a}{1-r}\right]$

$S_2 = \frac{2}{1-\frac{1}{3}} = 3, \qquad S_3 = \frac{3}{1-\frac{1}{4}} = 4,$

$S_n = \frac{n}{1-\frac{1}{n+1}} = (n+1)$

$\therefore \quad S_1^2 + S_2^2 + S_3^2 + ... + S_{2n-1}^2$
$= 2^2 + 3^2 + 4^2 + ... + (n+1)^2 + ... + (2n)^2$

NOTE THIS STEP :

$$\sum_{r=1}^{2n} r^2 - 1 = \frac{2n(2n+1)(4n+1)}{6} - 1^2$$

$$= \frac{n(2n+1)(4n+1) - 3}{3}$$

12. Since x_1, x_2, x_3 are in A.P. Therefore, let $x_1 = a-d, x_2 = a$ and $x_3 = a+d$ and x_1, x_2, x_3 are the roots of $x^3 - x^2 + \beta x + \gamma = 0$

We have $\Sigma\alpha = a - d + a + a + d = 1$ ....(1)

$\Sigma\alpha\beta = (a-d)\,a + a\,(a+d) + (a-d)\,(a+d) = \beta$ ....(2)

$\alpha\,\beta\,\gamma = (a-d)\,a\,(a+d) = -\gamma$ ....(3)

From (1), we get, $3a = 1 \Rightarrow a = 1/3$

From (2), we get, $3a^2 - d^2 = \beta$

$\Rightarrow 3\,(1/3)^2 - d^2 = \beta \quad \Rightarrow \quad 1/3 - \beta = d^2$

We know that $d^2 \geq 0 \ \forall \ d \in R$

$\Rightarrow \quad \frac{1}{3} - \beta \geq 0 \qquad \because d^2 \geq 0$

$\Rightarrow \quad \beta \leq \frac{1}{3} \Rightarrow \beta \in (-\infty, 1/3]$

From (3), $a\,(a^2 - d^2) = -\gamma$

$\Rightarrow \quad \frac{1}{3}\left(\frac{1}{9} - d^2\right) = -\gamma \Rightarrow \frac{1}{27} - \frac{1}{3}d^2 = -\gamma$

$\Rightarrow \quad \gamma + \frac{1}{27} = \frac{1}{3}d^2 \Rightarrow \gamma + \frac{1}{27} \geq 0$

$\Rightarrow \quad \gamma \geq -\frac{1}{27} \Rightarrow \gamma \in \left[-\frac{1}{27}, \infty\right)$

Hence $\beta \in (-\infty, 1/3)$ and $\gamma \in [-1/27, \infty]$

13. Solving the system of equations, $u+2v+3w=6$, $4u+5v+6w=12$ and $6u+9v=4$ we get $u=-1/3$, $v=2/3$, $w=5/3$

$\therefore \quad u+v+w=2,\ \frac{1}{u}+\frac{1}{v}+\frac{1}{w}=-\frac{9}{10}$

Let r be the common ratio of the G.P., a, b, c, d. Then $b=ar$, $c=ar^2$, $d=ar^3$.

Then the first equation

$$\left(\frac{1}{u}+\frac{1}{v}+\frac{1}{w}\right)x^2+[(b-c)^2+(c-a)^2+(d-b)^2]x+(u+v+w)=0$$

becomes

$$-\frac{9}{10}x^2+[(ar-ar^2)^2+(ar^2-a)^2+(ar^3-ar)^2]\,x+2=0$$

i.e., $9x^2-10a^2(1-r)^2[r^2+(r+1)^2+r^2(r+1)^2]x-20=0$

i.e., $9x^2-10a^2(1-r)^2(r^4+2r^3+3r^2+2r+1)x-20=0$

i.e., $9x^2-10a^2(1-r)^2(1+r+r^2)^2x-20=0$,

i.e., $9x^2-10a^2(1-r^3)^2x-20=0$ ....(1)

The second equation is,

$20x^2+10(a-ar^3)^2x-9=0$

i.e., $20x^2+10a^2(1-r^3)^2x-9=0$ ....(2)

Since (2) can be obtained by the substitution $x\to 1/x$, equations (1) and (2) have reciprocal roots.

14. Let $a-3d, a-d, a+d$ and $a+3d$ be any four consecutive terms of an A.P. with common difference 2d. $\because$ Terms of A.P. are integers, $2d$ is also an integer.

Hence $P=(2d)^4+(a-3d)(a-d)(a+d)(a+3d)$

$=16d^4+(a^2-9d^2)(a^2-d^2)=(a^2-5d^2)^2$

Now, $a^2-5d^2=a^2-9d^2+4d^2$

$=(a-3d)(a+3d)+(2d)^2=$ some integer

Thus, $P=$ square of an integer.

15. Given that $a_1, a_2, ...a_n$ are +ve real no's in G.P.

$$\therefore \left.\begin{aligned} a_1&=a\\ a_2&=ar\\ a_3&=ar^2\\ &\vdots\\ a_n&=ar^{n-1}\end{aligned}\right\}\quad \text{As } a_1,a_2,....a_n \text{ are +ve} \quad \therefore\ r>0$$

A_n is A.M. of $a_1, a_2, ..., a_n$

$$\therefore\ A_n=\frac{a_1+a_2+....+a_n}{n}=\frac{a+ar+....+ar^{n-1}}{n}$$

$$A_n=\frac{a(1-r^n)}{n(1-r)} \quad .... (1)\ (\text{For } r\neq 1)$$

G_n is G.M. of $a_1, a_2, ..., a_n$

$$\therefore\ G_n=\sqrt[n]{a_1,a_2...a_n}=\sqrt[n]{a.ar.ar^2....ar^{n-1}}$$

$$=\sqrt[n]{a^n,r^{\frac{n(n-1)}{2}}}=ar^{\frac{(n-1)}{2}}$$

$$G_n=ar^{\frac{(n-1)}{2}} \quad .... (2)\ (r\neq 1)$$

H_n is H.M. of $a_1, a_2, ..., a_n$

$$\therefore\ H_n=\frac{n}{\frac{1}{a_1}+\frac{1}{a_2}+....+\frac{1}{a_n}}=\frac{n}{\frac{1}{a}+\frac{1}{ar}+....+\frac{1}{ar^{n-1}}}$$

$$=\frac{n}{\frac{1}{a}\frac{\left(\frac{1}{r^n}-1\right)}{\left(\frac{1}{r}-1\right)}}=\frac{n}{\frac{1}{a}\left(\frac{1-r^n}{r^n}\right)\cdot\frac{r}{1-r}}$$

$$H_n=\frac{anr^{n-1}(1-r)}{(1-r^n)} \quad (r\neq 1) \quad ...(3)$$

We also observe that

$$A_nH_n=\frac{a(1-r^n)}{(n(1-r))}\times\frac{anr^{n-1}(1-r)}{(1-r^n)}=a^nr^{n-1}=G_n^2$$

$\therefore \quad A_nH_n=G_n^2$ ....(4)

$\therefore$ Now, G.M. of $G_1, G_2, ... G_n$ is

$G=\sqrt[n]{G_1\,G_2\,....G_n}$

$G=\sqrt[n]{\sqrt{A_1H_1}\sqrt{A_2H_2}...\sqrt{A_nH_n}}$

[Using equation (4)

$G=(A_1A_2....A_nH_1H_2....H_n)^{1/2n}$ ...(5)

If $r=1$ then

$A_n=G_n=H_n=a$

Also $A_nH_n=G_n^2$

$\therefore$ For $r=1$ also, equation (5) holds.

Hence we get,

$G=(A_1A_2....A_nH_1H_2....H_n)^{1/2n}$

16. Clearly $A_1+A_2=a+b$

$$\frac{1}{H_1}+\frac{1}{H_2}=\frac{1}{a}+\frac{1}{b}$$

$$\Rightarrow \frac{H_1+H_2}{H_1H_2}=\frac{a+b}{ab}=\frac{A_1+A_2}{G_1G_2}\Rightarrow\frac{G_1G_2}{H_1H_2}=\frac{A_1+A_2}{H_1+H_2}$$

$$\text{Also } \frac{1}{H_1}=\frac{1}{a}+\frac{1}{3}\left(\frac{1}{b}-\frac{1}{a}\right)\Rightarrow H_1=\frac{3ab}{2b+a}$$

$$\frac{1}{H_2}=\frac{1}{a}+\frac{2}{3}\left(\frac{1}{b}-\frac{1}{a}\right)\Rightarrow H_2=\frac{3ab}{2a+b}$$

$$\Rightarrow \frac{A_1+A_2}{H_1+H_2}=\frac{a+b}{3ab\left(\frac{1}{2b+a}+\frac{1}{2a+b}\right)}$$

$$=\frac{(2b+a)(2a+b)}{9ab}$$

17. Given that a, b, c are in A.P.

$\Rightarrow\ 2b=a+c$ ....(1)

and a^2, b^2, c^2 are in H.P.

$$\Rightarrow \frac{1}{b^2}-\frac{1}{a^2}=\frac{1}{c^2}-\frac{1}{b^2}$$

$$\Rightarrow \frac{(a-b)(a+b)}{b^2a^2}=\frac{(b-c)(b+c)}{b^2c^2}$$

$\Rightarrow ac^2+bc^2=a^2b+a^2c \quad [\because a-b=b-c]$

$\Rightarrow ac(c-a)+b(c-a)(c+a)=0$

$\Rightarrow (c-a)(ab+bc+ca)=0$

$\Rightarrow$ either $c-a=0$ or $ab+bc+ca=0$

$\Rightarrow$ either $c = a$ or $(a+c)\,b + ca = 0$
and then from (i) $2b^2 + ca = 0$

Either $a = b = c$ or $b^2 = a\left(\frac{-c}{2}\right)$

i.e. a, b, – c/2 are in G.P. **Hence Proved.**

18. $a_n = \frac{3}{4} - \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + ... + (-1)^{n-1}\left(\frac{3}{4}\right)^n$

$$= \frac{\frac{3}{4}\left(1-\left(-\frac{3}{4}\right)^n\right)}{1+\frac{3}{4}} = \frac{3}{7}\left(1-\left(-\frac{3}{4}\right)^n\right)$$

$b_n = 1 - a_n$ and $b_n > a_n \ \forall\ n \geq n_0$

$\therefore$ $1 - a_n > a_n \Rightarrow 2a_n < 1$

$\Rightarrow \frac{6}{7}\left[1-\left(-\frac{3}{4}\right)^n\right] < 1 \Rightarrow -\left(-\frac{3}{4}\right)^n < \frac{1}{6}$

$\Rightarrow$ $(-3)^{n+1} < 2^{2n-1}$

For n to be even, inequality always holds. For n to be odd, it holds for $n \geq 7$.

$\therefore$ The least natural no., for which it holds is 6
($\because$ it holds for every even natural no.)

G. Comprehension Based Questions

1. **(b)** $V_1 + V_2 + ... + V_n = \sum_{r=1}^{n} V_r = \sum_{r=1}^{n}\left(r^3 - \frac{r^2}{2} + \frac{r}{2}\right)$

$$= \Sigma n^3 - \frac{\Sigma n^2}{2} + \frac{\Sigma n}{2}$$

$$= \frac{n^2(n+1)^2}{4} - \frac{n(n+1)\,(2n+1)}{12} + \frac{n(n+1)}{4}$$

$$= \frac{n(n+1)}{4}\left[n(n+1) - \frac{2n+1}{3} + 1\right]$$

$$= \frac{n(n+1)\,(3n^2+n+2)}{12}$$

2. **(d)** $T_r = V_{r+1} - V_r - 2$

$$= \left[(r+1)^3 - \frac{(r+1)^2}{2} + \frac{r+1}{2}\right] - \left[r^3 - \frac{r^2}{2} + \frac{r}{2}\right] - 2$$

$= 3r^2 + 2r + 1$

$T_r = (r+1)(3r-1)$

For each r, T_r has two different factors other than 1 and itself.

$\therefore$ T_r is always a composite number.

3. **(b)** $\therefore Q_{r+1} - Q_r = T_{r+2} - T_{r+1} - (T_{r+1} - T_r)$

$= T_{r+2} - 2T_{r+1} + T_r$

$= (r+3)(3r+5) - 2(r+2)(3r+2) + (r+1)(3r-1)$

$\because$ $Q_{r+1} - Q_r = 6(r+1) + 5 - 6r - 5 = 6$ (constant)

$\therefore$ Q_1, Q_2, Q_3, ... are in AP with common difference 6.

4. **(c)** Given $A_1 = \frac{a+b}{2}, G_1 = \sqrt{ab},\ H_1 = \frac{2ab}{a+b}$

also $A_n = \frac{A_{n-1} + H_{n-1}}{2}, G_n = \sqrt{A_{n-1}H_{n-1}}$

$H_n = \frac{2A_{n-1}H_{n-1}}{A_{n-1} + H_{n-1}}$

$\Rightarrow G_n^2 = A_nH_n \Rightarrow A_nH_n = A_{n-1}H_{n-1}$

Similarly we can prove

$A_nH_n = A_{n-1}H_{n-1} = A_{n-2}H_{n-2} = ... = A_1H_1$

$\Rightarrow$ $A_nH_n = ab$ $\therefore$

$\therefore$ $G_1^2 = G_2^2 = G_3^2$ .... $= ab$

$\Rightarrow$ $G_1 = G_2 = G_3$ .... $= \sqrt{ab}$

5. **(a)** We have $A_n = \frac{A_{n-1} + H_{n-1}}{2}$

$\therefore$ $A_n - A_{n-1} = \frac{A_{n-1} + H_{n-1}}{2} - A_{n-1}$

$= \frac{H_{n-1} - A_{n-1}}{2} < 0$ ($\because A_{n-1} > H_{n-1}$)

$\Rightarrow$ $A_n < A_{n-1}$ or $A_{n-1} > A_n$

$\therefore$ We can conclude that $A_1 > A_2 > A_3 > ....$

6. **(b)** We have $A_nH_n = ab \Rightarrow H_n = \frac{ab}{A_n}$

$\because$ $\frac{1}{A_{n-1}} < \frac{1}{A_n} \Rightarrow H_{n-1} < H_n$ $\therefore H_1 < H_2 < H_3 < ....$

H. Assertion & Reason Type Questions

1. **(c)** Given a_1, a_2, a_3, a_4 are in GP.

Then b_1, b_2, b_3, b_4 are the numbers

$a_1, a_1 + a_2, a_1 + a_2 + a_3,\ a_1 + a_2 + a_3 + a_4$

or $a, a + ar,\ a + ar + ar^2,\ a + ar + ar^2 + ar^3$

Clearly above numbers are neither in AP nor in G.P. and hence statement 1 is true.

Also $\frac{1}{a}, \frac{1}{a+ar}, \frac{1}{a+ar+ar^2}, \frac{1}{a+ar+ar^2+ar^3}$ are not in A.P. $\therefore b_1, b_2, b_3, b_4$ are not in H.P.

$\therefore$ Statement 2 is false.

I. Integer Value Correct Type

1. **(3)** Using $S_\infty = \frac{a}{1-r}$, we get

$$S_k = \begin{cases} \dfrac{\frac{k-1}{k!}}{1-\frac{1}{k!}}, & k \neq 1 \\ 0, & k = 1 \\ \dfrac{1}{(k-1)!}, & k \geq 2 \end{cases}$$

Now $\sum_{k=1}^{100}|(k^2-3k+1)S_k| = \sum_{k=2}^{100}|(k^2-3k+1)|\frac{1}{(k-1)!}$

$=|-1|+\sum_{k=3}^{100}\frac{(k^2-1)+1-3(k-1)-2}{(k-1)!}$ as $k^2-3k+1>0\,\forall k\geq 3$

$= 1+\sum_{k=3}^{100}\left(\frac{1}{(k-3)!}-\frac{1}{(k-1)!}\right)$

$= 1+\left(1-\frac{1}{2!}\right)+\left(\frac{1}{1!}-\frac{1}{3!}\right)+\left(\frac{1}{2!}-\frac{1}{4!}\right)+....$

$....+\left(\frac{1}{96!}-\frac{1}{98!}\right)+\left(\frac{1}{97!}-\frac{1}{99!}\right)$

$= 3-\frac{1}{98!}-\frac{1}{99!}=3-\frac{9900}{100!}-\frac{100}{100!}=3-\frac{10000}{100!}=3-\frac{(100)^2}{100!}$

$\therefore\ \frac{100^2}{100!}+3\sum_{k=1}^{100}\left|\left(k^2-3k+1\right)S_k\right|=3.$

2. **(0)** Given that $a_k=2a_{k-1}-a_{k-2}$

$\Rightarrow \frac{a_{k-2}+a_k}{2}=a_{k-1},\ 3\leq k\leq 11$

$\Rightarrow a_1, a_2, a_3, ..., a_{11}$ are in AP.

If a is the first term and D the commen difference then

$a_1^2+a_2^2+...+a_{11}^2=990$

$\Rightarrow 11a^2+d^2\left(1^2+2^2+...+10^2\right)+2ad\left(1+2+...+10\right)$

$=990$

$\Rightarrow 11a^2+\frac{10\times 11\times 21}{6}d^2+2ad\times\frac{10\times 11}{2}=990$

$\Rightarrow a^2+35d^2+150d=90$

Using $a = 15$, we get

$35d^2+150d+135=0$ or $7d^2+30d+27=0$

$\Rightarrow (d+3)(7d+9)=0 \Rightarrow d=-3$ or $-9/7$

then $a_2=15-3=12$ or $15-\frac{9}{7}=\frac{96}{7}>\frac{27}{2}$

$\therefore$ d $\neq -9/7$

Hence $\frac{a_1+a_2+...+a_{11}}{11}=\frac{\frac{11}{2}\left[2\times 15+10(-3)\right]}{11}=0$

3. **(9)**

We have $\frac{S_m}{S_n}=\frac{S_{5n}}{S_n}=\frac{\frac{5n}{2}[2\times 3+(5n-1)d]}{\frac{n}{2}[6+(n-1)d]}$

$=\frac{5[(6-d)+5nd]}{(6-d)+nd]}$

which will be independent of n if $d = 6$ or $d = 0$

For a proper A.P. we take $d = 6$

then $a_2 = 3 + 6 = 9$

4. **(5)** Let k, $k + 1$ be removed from pack.

$\therefore (1+2+3+...+n)-(k+k+1)=1224$

$\frac{n(n+1)}{2}-2k=1225$

$k=\frac{n(n+1)-2450}{4}$

for $n = 50$, $k = 25$

$\therefore\ k - 20 = 5$

5. **(4)** $\because a, b, c$ are in G.P

$\therefore b = ar$ and $c = ar^2$

Also $\frac{b}{a}$ is an integer

$\Rightarrow r$ is an integer

$\because$ A.M. of a, b, c is $b + 2$

$\Rightarrow \frac{a+b+c}{3}=b+2$

$\Rightarrow\ a+ar+ar^2=3ar+6$

$\Rightarrow\ a\left(r^2-2r+1\right)=6$

$\Rightarrow\ a(r-1)^2=6$

$\because a$ and r are integers

$\therefore$ The only possible values of a and r can be 6 and 2 respectively.

Then $\frac{a^2+a-14}{a+1}=\frac{36+6-14}{6+1}=\frac{28}{7}=4$

6. **(9)** $\frac{\frac{7}{2}[2a+6d]}{\frac{11}{2}[2a+10d]}=\frac{6}{11}\Rightarrow a=9d$

$a_7 = a + 6d = 15d$

$\because\ 130<15d<140\Rightarrow d=9$

($\because$ All terms are natural numbers $\therefore$ $d\in N$)

7. **(8)** In expansion of $(1+x)(1+x^2)(1+x^3)....(1+x^{100})$

x^9 can be found in the following ways

$x^9, x^{1+8}, x^{2+7}, x^{3+6}, x^{4+5}, x^{1+2+6}, x^{1+3+5}, x^{2+3+4}$

The coefficient of x^9 in each of the above 8 cases is 1.

$\therefore$ Required coefficient = 8.

Section-B JEE Main/ AIEEE

1. **(b)** $1, \log_9(3^{1-x}+2), \log_3(4.3^x-1)$ are in A.P.

$\Rightarrow 2\log_9(3^{1-x}+2) = 1+\log_3(4.3^x-1)$

$\Rightarrow \log_3(3^{1-x}+2) = \log_3 3+\log_3(4.3^x-1)$

$\Rightarrow \log_3(3^{1-x}+2) = \log_3[3(4.3^x-1)]$

$\Rightarrow 3^{1-x}+2 = 3(4.3^x-1)$

$\Rightarrow 3.3^{-x}+2 = 12.3^x-3.$

Put $3^x = t$

$\Rightarrow \frac{3}{t}+2 = 12t-3$ or $12t^2-5t-3=0;$

Hence $t=-\frac{1}{3},\frac{3}{4} \Rightarrow 3^x = \frac{3}{4}$ (as $3^x \neq -ve$)

$\Rightarrow x = \log_3\left(\frac{3}{4}\right)$ or $x=\log_3 3-\log_3 4$

$\Rightarrow x = 1-\log_3 4$

2. **(d)** $l = AR^{p-1} \Rightarrow \log l = \log A+(p-1)\log R$

$m = AR^{q-1} \Rightarrow \log m = \log A+(q-1)\log R$

$n = AR^{r-1} \Rightarrow \log n = \log A+(r-1)\log R$

Now,

$$\begin{vmatrix} \log l & p & 1 \\ \log m & q & 1 \\ \log n & r & 1 \end{vmatrix} = \begin{vmatrix} \log A+(p-1)\log R & p & 1 \\ \log A+(q-1)\log R & q & 1 \\ \log A+(r-1)\log R & r & 1 \end{vmatrix}$$

Operating $C_1-(\log R)C_2+(\log R-\log A)C_3$

$$= \begin{vmatrix} 0 & p & 1 \\ 0 & q & 1 \\ 0 & r & 1 \end{vmatrix} = 0$$

3. **(b)** The product is $P = 2^{1/4}.2^{2/8}.2^{3/16}$.........

$= 2^{1/4+2/8+3/16+........\infty}$

Now let $S = \frac{1}{4}+\frac{2}{8}+\frac{3}{16}+........\infty$ ......(1)

$\frac{1}{2}S = \frac{1}{8}+\frac{2}{16}+........\infty$ ......(2)

Subtracting (2) from (1)

$\Rightarrow \frac{1}{2}S = \frac{1}{4}+\frac{1}{8}+\frac{1}{16}+........\infty$

or $\frac{1}{2}S = \frac{1/4}{1-1/2} = \frac{1}{2} \Rightarrow S=1$

$\therefore P = 2^S = 2$

4. **(b)** $ar^4 = 2$

$a\times ar\times ar^2\times ar^3\times ar^4\times ar^5\times ar^6\times ar^7\times ar^8$

$= a^9r^{36} = (ar^4)^9 = 2^9 = 512$

5. **(b)** Let a = first term of G.P. and r = common ratio of G.P.; Then G.P. is a, ar, ar^2

Given $S_\infty = 20 \Rightarrow \frac{a}{1-r} = 20 \Rightarrow a = 20(1-r)$... (i)

Also $a^2+a^2r^2+a^2r^4+...$ to $\infty = 100$

$\Rightarrow \frac{a^2}{1-r^2} = 100 \Rightarrow a^2 = 100(1-r)(1+r)$... (ii)

From (i), $a^2 = 400(1-r)^2$;

From (ii), we get $100(1-r)(1+r) = 400(1-r)^2$

$\Rightarrow 1+r = 4-4r \Rightarrow 5r=3 \Rightarrow r=3/5.$

6. **(a)** $1^3-2^3+3^3-4^3+........+9^3$

$= 1^3+2^3+3^3+.......+9^3-2(2^3+4^3+6^3+8^3)$

$= \left[\frac{9\times 10}{2}\right]^2 - 2.2^3\left[1^3+2^3+3^3+4^3\right]$

$= (45)^2-16.\left[\frac{4\times 5}{2}\right]^2 = 2025-1600 = 425$

7. **(a)** $\frac{1}{1.2}-\frac{1}{2.3}+\frac{1}{3.4}$...............$\infty$

$|T_n| = \frac{1}{n(n+1)} = \left(\frac{1}{n}-\frac{1}{n+1}\right)$

$S = T_1-T_2+T_3-T_4+T_5$............∞

$= \left(\frac{1}{1}-\frac{1}{2}\right)-\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)-\left(\frac{1}{4}-\frac{1}{5}\right)$.....

$= 1-2\left[\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}................\infty\right]$

$= 1-2[-\log(1+1)+1] = 2\log 2-1 = \log\left(\frac{4}{e}\right).$

8. **(d)** $S_n = \frac{1}{^nC_0}+\frac{1}{^nC_1}+\frac{1}{^nC_2}+....+\frac{1}{^nC_n}$

$t_n = \frac{0}{^nC_0}+\frac{1}{^nC_1}+\frac{2}{^nC_2}+....+\frac{n}{^nC_n}$

$t_n = \frac{n}{^nC_n}+\frac{n-1}{^nC_{n-1}}+\frac{n-2}{^nC_{n-2}}+....+\frac{0}{^nC_0}$

Add, $2t_n = (n)\left[\frac{1}{^nC_0}+\frac{1}{^nC_1}+....\frac{1}{^nC_n}\right] = nS_n$

$\therefore \frac{t_n}{S_n} = \frac{n}{2}$

9. (d) $T_m = a + (m-1)d = \frac{1}{n}$ ....(1)

$T_n = a + (n-1)d = \frac{1}{m}$ ....(2)

$(1) - (2) \Rightarrow (m-n)d = \frac{1}{n} - \frac{1}{m} \Rightarrow d = \frac{1}{mn}$

From (1) $a = \frac{1}{mn} \Rightarrow a - d = 0$

10. (b) If n is odd, the required sum is

$1^2 + 2.2^2 + 3^2 + 2.4^2 + ... + 2.(n-1)^2 + n^2$

$= \frac{(n-1)(n-1+1)^2}{2} + n^2$

[$\because$ $(n-1)$ is even

$\therefore$ using given formula for the sum of $(n-1)$ terms.]

$= \left(\frac{n-1}{2} + 1\right)n^2 = \frac{n^2(n+1)}{2}$

11. (b) We know that $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + ...$

and $e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + ...$

$\therefore e + e^{-1} = 2\left[1 + \frac{1}{2!} + \frac{1}{4!} + ...\right]$

$\therefore \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + ... = \frac{e + e^{-1}}{2} - 1$

$= \frac{e^2 + 1 - 2e}{2e} = \frac{(e-1)^2}{2e}$

12. (c) Given ${}^mC_{r-1},\ {}^mC_r,\ {}^mC_{r+1}$ are in A.P.

$2\,{}^mC_r = {}^mC_{r-1} + {}^mC_{r+1}$

$\Rightarrow 2 = \frac{{}^mC_{r-1}}{{}^mC_r} + \frac{{}^mC_{r+1}}{{}^mC_r} = \frac{r}{m-r+1} + \frac{m-r}{r+1}$

$\Rightarrow m^2 - m(4r+1) + 4r^2 - 2 = 0$.

13. (d) $x = \sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$ $\quad a = 1 - \frac{1}{x}$

$y = \sum_{n=0}^{\infty} b^n = \frac{1}{1-b}$ $\quad b = 1 - \frac{1}{y}$

$z = \sum_{n=0}^{\infty} c^n = \frac{1}{1-c}$ $\quad c = 1 - \frac{1}{z}$

a, b, c are in A.P. OR $2b = a + c$

$2\left(1 - \frac{1}{y}\right) = 1 - \frac{1}{x} + 1 - \frac{1}{y}$

$\frac{2}{y} = \frac{1}{x} + \frac{1}{z} \Rightarrow x, y, z$ are in H.P.

14. (d) $\frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!}$............

Putting $x = \frac{1}{2}$ we get

$1 + \frac{1}{4.2!} + \frac{1}{16.4!} + \frac{1}{64.6!} + ...$

$\infty = \frac{e^{\frac{1}{2}} + e^{\frac{-1}{2}}}{2} = \frac{\sqrt{e} + \frac{1}{\sqrt{e}}}{2} = \frac{e+1}{2\sqrt{e}}$

15. (d) $\frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2} \Rightarrow \frac{2a_1 + (p-1)d}{2a_1 + (q-1)d} = \frac{p}{q}$

$\frac{a_1 + \left(\frac{p-1}{2}\right)d}{a_1 + \left(\frac{q-1}{2}\right)d} = \frac{p}{q}$

For $\frac{a_6}{a_{21}}$, $p = 11, q = 41 \Rightarrow \frac{a_6}{a_{21}} = \frac{11}{41}$

16. (d) $\frac{1}{a_2} - \frac{1}{a_1} = \frac{1}{a_3} - \frac{1}{a_2} = ... = \frac{1}{a_n} - \frac{1}{a_{n-1}} = d$ (say)

Then $a_1a_2 = \frac{a_1 - a_2}{d}$, $a_2a_3 = \frac{a_2 - a_3}{d}$,

..........., $a_{n-1}a_n = \frac{a_{n-1} - a_n}{d}$

$\therefore$ $a_1a_2 + a_2a_3 + ... + a_{n-1}a_n$

$= \frac{a_1 - a_2}{d} + \frac{a_2 - a_3}{d} + ... + \frac{a_{n-1} - a_n}{d}$

$= \frac{1}{d}[a_1 - a_2 + a_2 - a_3 + ... + a_{n-1} - a_n] = \frac{a_1 - a_n}{d}$

Also, $\frac{1}{a_n} = \frac{1}{a_1} + (n-1)d$

$\Rightarrow \frac{a_1 - a_n}{a_1a_n} = (n-1)d \Rightarrow \frac{a_1 - a_n}{d} = (n-1)a_1a_n$

Which is the required result.

17. **(d)** We know that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...\infty$

Put x = – 1

$\therefore \quad e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}........\infty$

$\therefore \quad e^{-1} = \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}........\infty$

18. **(b)** Let the series a, ar, ar^2, ... are in geometric progression.
given, a = ar + ar^2

$\Rightarrow \quad 1 = r + r^2 \quad \Rightarrow r^2 + r - 1 = 0$

$\Rightarrow \quad r = \frac{-1 \pm \sqrt{1 - 4 \times -1}}{2} \Rightarrow \quad r = \frac{-1 \pm \sqrt{5}}{2}$

$\Rightarrow \quad r = \frac{\sqrt{5} - 1}{2}$ [$\because$ terms of G.P. are positive

$\therefore$ r should be positive]

19. **(b)** As per question,

a + ar = 12 ...(1)

$ar^2 + ar^3 = 48$ ...(2)

$\Rightarrow \quad \frac{ar^2(1+r)}{a(1+r)} = \frac{48}{12} \Rightarrow r^2 = 4, \Rightarrow r = -2$

($\because$ terms are = + ve and –ve alternately)

$\Rightarrow \quad a = -12$

20. **(a)** We have

$S = 1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \frac{14}{3^4} + ...\infty$ ....(1)

Multiplying both sides by $\frac{1}{3}$ we get

$\frac{1}{3} S = \frac{1}{3} + \frac{2}{3^2} + \frac{6}{3^3} + \frac{10}{3^4} + ...\infty$ ....(2)

Subtracting eqn. (2) from eqn. (1) we get

$\frac{2}{3} S = 1 + \frac{1}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...\infty$

$\Rightarrow \quad \frac{2}{3} S = \frac{4}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \frac{4}{3^4} + ...\infty$

$\Rightarrow \quad \frac{2}{3} S = \frac{\frac{4}{3}}{1 - \frac{1}{3}} = \frac{4}{3} \times \frac{3}{2} \Rightarrow \quad S = 3$

21. **(a)** Till 10^{th} minute number of counted notes = 1500

$3000 = \frac{n}{2}[2 \times 148 + (n-1)(-2)] = n[148 - n + 1]$

$n^2 - 149n + 3000 = 0$

$\Rightarrow n = 125, 24$

But $n = 125$ is not possible

$\therefore$ total time = 24 + 10 = 34 minutes.

22. **(c)** Let required number of months = n

$\therefore 200 \times 3 + (240 + 280 + 320 + ... + (n-3)^{th} \text{ term}) = 11040$

$\Rightarrow \quad \frac{n-3}{2}[2 \times 240 + (n-4) \times 40] = 11040 - 600$

$\Rightarrow \quad (n-3)[240 + 20n - 80] = 10440$

$\Rightarrow \quad (n-3)(20n + 160) = 10440$

$\Rightarrow \quad (n-3)(n+8) = 522$

$\Rightarrow \quad n^2 + 5n - 546 = 0 \Rightarrow \quad (n+26)(n-21) = 0$

$\therefore \; n = 21$

23. **(b)** nth term of the given series

$= T_n = (n-1)^2 + (n-1)n + n^2$

$= \frac{\left((n-1)^3 - n^3\right)}{(n-1) - n} = n^3 - (n-1)^3$

$\Rightarrow S_n = \sum_{k=1}^{n}\left[k^3 - (k-1)^3\right] \Rightarrow 8000 = n^3$

$\Rightarrow \quad n = 20$ which is a natural number.

Now, put n = 1,2,3,....20

$T_1 = 1^3 - 0^3$

$T_2 = 2^3 - 1^3$

$\vdots$

$T_{20} = 20^3 - 19^3$

Now, $T_1 + T_2 + --- + T_{20} = S_{20}$

$\Rightarrow \quad S_{20} = 20^3 - 0^3 = 8000$

Hence, both the given statements are true and statement 2 supports statement 1.

24. **(c)** Given sequence can be written as

$\frac{7}{10} + \frac{77}{100} + \frac{777}{10^3} + ... +$ up to 20 terms

$= 7\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{10^3} + ... + \text{up to 20 terms}\right]$

Multiply and divide by 9

$= \frac{7}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + ...\text{up to 20 terms}\right]$

$= \frac{7}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + ...\text{up to 20 terms}\right]$

$= \frac{7}{9}\left[20 - \frac{\frac{1}{10}\left(1 - \left(\frac{1}{10}\right)^{20}\right)}{1 - \frac{1}{10}}\right]$

$= \frac{7}{9}\left[\frac{179}{9} + \frac{1}{9}\left(\frac{1}{10}\right)^{20}\right] = \frac{7}{81}[179 + (10)^{-20}]$

25. (a) Let $10^9 + 2.(11)(10)^8 + 3(11)^2(10)^7 + ... + 10(11)^9 = k(10)^9$

Let $x = 10^9 + 2.(11)(10)^8 + 3(11)^2(10)^7 + ... + 10(11)^9$

Multiplied by $\frac{11}{10}$ on both the sides

$$\frac{11}{10}x = 11.10^8 + 2.(11)^2.(10)^7 + ... + 9(11)^9 + 11^{10}$$

$$x\left(1-\frac{11}{10}\right) = 10^9 + 11(10)^8 + 11^2\times(10)^7 + ... + 11^9 - 11^{10}$$

$$\Rightarrow -\frac{x}{10} = 10^9\left[\frac{\left(\frac{11}{10}\right)^{10}-1}{\frac{11}{10}-1}\right] - 11^{10}$$

$$\Rightarrow -\frac{x}{10} = (11^{10} - 10^{10}) - 11^{10} = -10^{10}$$

$\Rightarrow x = 10^{11} = k.10^9$ Given $\Rightarrow k = 100$

26. (b) Let a, ar, ar^2 are in G.P.
According to the question
$a, 2ar, ar^2$ are in A.P.
$\Rightarrow 2\times 2ar = a + ar^2$
$\Rightarrow 4r = 1 + r^2 \Rightarrow r^2 - 4r + 1 = 0$

$$r = \frac{4\pm\sqrt{16-4}}{2} = 2\pm\sqrt{3}$$

Since $r > 1$

$\therefore$ $r = 2-\sqrt{3}$ is rejected

Hence, $r = 2+\sqrt{3}$

27. (d) n^{th} term of series $= \frac{\left[\frac{n(n+1)}{2}\right]^2}{n^2} = \frac{1}{4}(n+1)^2$

Sum of n term $= \Sigma\frac{1}{4}(n+1)^2$

$$= \frac{1}{4}\left[\Sigma n^2 + 2\Sigma n + n\right]$$

$$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2} + n\right]$$

Sum of 9 terms

$$= \frac{1}{4}\left[\frac{9\times10\times19}{6} + \frac{18\times10}{2} + 9\right] = \frac{384}{4} = 96$$

28. (d) $m = \frac{l+n}{2}$ and common ratio of G.P. $= r = \left(\frac{n}{l}\right)^{\frac{1}{4}}$

$\therefore$ $G_1 = l^{3/4}n^{1/4}, G_2 = l^{1/2}n^{1/2}, G_3 = l^{1/4}n^{3/4}$

$G_1^4 + 2G_2^4 + G_3^4 = l^3n + 2l^2n^2 + ln^3$
$= ln(l+n)^2$
$= ln\times 2m^2$
$= 4lm^2n$

29. (d) Let the GP be a, ar and ar^2 then $a = A + d$; $ar = A + 4d$; $ar^2 = A + 8d$

$$\Rightarrow \frac{ar^2 - ar}{ar - a} = \frac{(A+8d)-(A+4d)}{(A+4d)-(A+d)}$$

$$r = \frac{4}{3}$$

30. (d) $\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 ... + \left(\frac{44}{5}\right)^2$

$$S = \frac{16}{25}\left(2^2 + 3^2 + 4^2 + ... + 11^2\right)$$

$$= \frac{16}{25}\left(\frac{11(11+1)(22+1)}{6} - 1\right)$$

$$= \frac{16}{25}\times505 = \frac{16}{5}\times101$$

$$\Rightarrow \frac{16}{5}m = \frac{16}{5}\times101$$

$\Rightarrow m = 101.$